SOME SIMPLE ACTIVE FILTERS FOR LOW FREQUENCIES

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The growing interest shown during recent years in electrical measurements at very low frequencies has led to a demand for special low-frequency filters. Coils, which are commonly used in filters for higher frequencies, have certain disadvantages at low frequencies. In this article the authors describe some circuits in which the use of coils is avoided.

Introduction

In radio engineering and in other branches of electrotechnology it is the practice to give circuits specified frequency-dependent characteristics by using capacitors and coils. These two circuit elements have properties that are to a certain extent opposed to one another. When an alternating voltage is applied, the current flowing in a capacitor leads the voltage in phase, whereas in a coil the current lags in phase. Again, in a capacitor the reactance decreases with rising frequency, whereas in a coil it increases.

Among the passive electric networks composed of combinations of capacitors and coils, the electric filters, which are used for separating signals with certain frequencies from signals with other frequencies, form an important subgroup. For this purpose the use of capacitors *and* coils is not strictly necessary. A filter can be built using only one of these elements in combination with resistors. By combining coils and capacitors, however, it is easy to produce a much sharper separation between wanted and unwanted signals.

One of the simplest and most familiar passive filter networks is the parallel resonant circuit (here referred to as an LC circuit) which can be considered as formed from the parallel arrangement of a capacitor, a coil (both loss-free) and a resistor (see *fig. 1*). In such a circuit the modulus of the impedance is maximum at the resonant frequency

$$\omega_0 = 1/\sqrt{LC}\,. \qquad (1)$$

For any given value ω of the frequency, the impedance can be written in the form:

$$Z(\omega) = \frac{R}{1 + \mathrm{j}\beta Q}, \qquad (2)$$

where β is an abbreviated notation for $\omega/\omega_0 - \omega_0/\omega$. If $|\omega - \omega_0|$ is small compared with ω_0, we can write β in the form:

$$\beta = \frac{2(\omega - \omega_0)}{\omega_0}.$$

In this case, then, β is twice the "relative detuning".

In equation (2) Q is the figure of merit, here given by:

$$Q = \frac{R}{\omega_0 L} = \omega_0 CR\,. \qquad (3)$$

The modulus of the impedance Z, plotted as a function of frequency, has the form of the familiar resonance curve (fig. 1). The curve is narrower, i.e. the LC circuit more selective, the higher the value of Q. The difference between the two frequencies at which $|Z|$ is a factor of $\sqrt{2}$ smaller than the maximum value is normally referred to as the bandwidth.

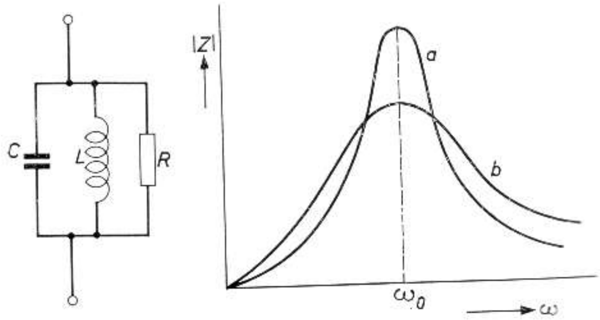

Fig. 1. Left: resonant circuit consisting of a coil, an inductance, a capacitance and a resistance in parallel LC circuit. Right: frequency characteristic (resonance curve) for two LC circuits. The resonant frequency is ω_0. The network to which curve a relates has a higher figure of merit Q.

Filters for very low frequencies

In recent years a growing interest has been shown in electrical measurements at very low frequencies. In the fields of physics and chemistry, for instance,

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measurements are frequently carried out with intermittent light; examples are measurements of photoconductivity and of light adsorption in liquids and crystals. The periodic interruption of the light makes it possible to use AC amplifiers for this purpose, which are easier to build and to operate than DC amplifiers. Measurements at very low frequencies are also important in various electro-medical applications, particularly in cardiography, encephalography and myography.

In some of these cases, filters are required which reduce interference to the minimum at the low frequencies involved. Sometimes, too, filters are needed for separating signals of different very low frequencies. This is notably the case in encephalography, where it is occasionally necessary to perform a harmonic analysis of the cerebral voltages, the presence or absence of voltages of certain frequencies having a bearing on the diagnosis of epilepsy. To give some idea of the resolving power required in such a case, it may be mentioned that filters are sometimes needed which have a bandwidth of 1 c/s at a centre frequency of some tens of cycles per second.

To build filters for lower frequencies it is necessary, as can be deduced from eq. (1), to use capacitors of higher capacitance and/or coils of higher inductance. As regards the capacitors, this does not as a rule present insuperable difficulties; the capacitance of a capacitor contained in a given volume can be increased by enlarging the surface area of the plates and by making the dielectric thinner. The construction of coils having a very high inductance involves considerable, and often intractable problems. Coils of this kind always contain a core, which may consist of iron alloy lamellae or of ferroxcube. This makes high-inductance coils relatively bulky and heavy, as also does the larger number of turns required to produce a higher inductance.

Apart from the drawbacks of large bulk and/or weight, there are various other disadvantages attached to the use of high-inductance coils. As a rule they are expensive, because of the large amount of material used, and it takes much more time to produce them. Such a coil, once made, is not very "flexible", that is to say its properties are difficult to modify. In most cases, indeed, it is not practicable to change the inductance value. This, together with the unavoidable variation during manufacture, can make it difficult to produce a coil with an accurately specified inductance. Another drawback is that the properties of coils of this kind tend to vary more markedly with temperature than those of most other circuit elements. Finally, problems can also arise from the necessity of screening the coils against stray magnetic fields. At low frequencies ferromagnetic cans have to be used for this purpose, possibly combined with one or more cans made of a material possessing good electrical conductivity. This again puts up the weight and the price.

The above-mentioned objections to the use of large coils have resulted in the development of circuits which contain no coils, but have the same properties as circuits that do. A familiar example is the feedback amplifier incorporating in the feedback loop a twin-T filter composed of capacitors and resistors [1]). The characteristics of a network of this kind correspond to those of an LC circuit. Alignment, however, is somewhat laborious, particularly where several amplifiers tuned to different frequencies are to be connected in cascade.

The object of this article is to draw attention to a type of filter with which the same or even better results can often be achieved, and which is simpler to align than the above-mentioned feedback amplifier. This type of filter has in fact long been known; here, however, after giving a brief outline of its principles, we shall consider some of the less familiar aspects. In particular it will be shown that various measures can be taken to achieve considerable precision with filters of this type and some lesser known applications will be touched upon [2]).

Principle of the circuit

Our starting point is a resistance-coupled amplifier stage, as represented by the diagram in *fig. 2*.

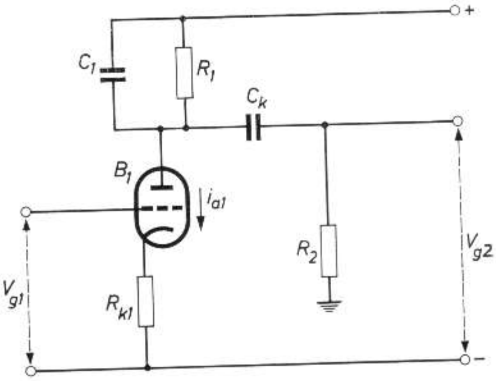

Fig. 2. Amplifying stage with resistance coupling.

[1]) A filter of this kind is described by G. Klein and J.J. Zaalberg van Zelst in: A low-frequency oscillator with very low distortion under non-linear loading, Philips tech. Rev. **25**, 22-30, 1963/64 (No. 1), p. 27.

[2]) A special application is described by K. Teer, Audibility of phase errors, Philips tech. Rev. **25**, 176-178, 1963/64 (No. 6/7).

Connected in parallel with the resistor R_1 in the anode circuit of the triode B_1 is a capacitor C_1, and the output voltage V_{g2} [3]) of the anode is taken off via a capacitance-resistance coupling C_k-R_2. It can readily be shown that the relation between the anode alternating current i_{a1} of B_1 and the voltage V_{g2} is:

$$\frac{V_{g2}}{i_{a1}}=\frac{-j\omega R_1R_2C_k}{1+j\omega(R_1C_1+R_1C_k+R_2C_k)-\omega^2R_1R_2C_1C_k}. \quad (4)$$

The same relation between V_{g2} and i_{a1} would have been found if we had incorporated in the anode network of B_1 an LC circuit consisting of a parallel arrangement of a resistance R', an inductance L' and a capacitance C' (see *fig. 3*). These three elements

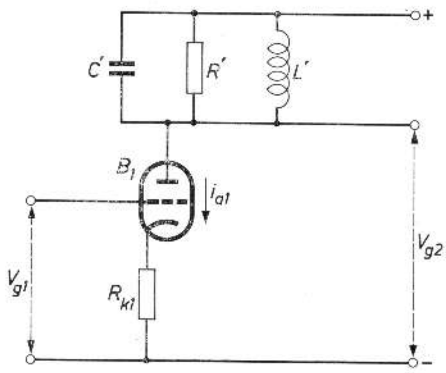

Fig. 3. With appropriate values of L', C' and R' this circuit is equivalent to fig. 2. The figure of merit is in that case very low.

would then have to have the following magnitudes:

$$R'=\frac{R_1R_2C_k}{R_1C_1+R_1C_k+R_2C_k}, \quad (5)$$

$$L'=R_1R_2C_k, \quad (6)$$

$$C'=C_1. \quad (7)$$

The resonant frequency of this equivalent circuit would thus be:

$$\omega_0=\frac{1}{\sqrt{L'C'}}=\frac{1}{\sqrt{R_1R_2C_1C_k}}, \quad (8)$$

and the figure of merit:

$$Q=\frac{R'}{\omega_0L'}=\frac{\sqrt{R_1R_2C_1C_k}}{R_1C_1+R_1C_k+R_2C_k}. \quad (9)$$

The quotient of V_{g2} and i_{a1} can then be written in the form:

$$\frac{V_{g2}}{i_{a1}}=\frac{-R'}{1+j\beta Q}. \quad (10)$$

If the resistance R_{k1} in the cathode lead of B_1 is large compared with the reciprocal of the transconductance, and if, moreover, the amplification factor of B_1 is very high, then $i_{a1}=V_{g1}/R_{k1}$, and eq. (10) becomes:

$$V_{g2}=\frac{-R'}{R_{k1}(1+j\beta Q)}V_{g1}. \quad (11)$$

This analogy with LC circuits is also found with normal amplifying stages. As a rule C_1 is then made up of the anode capacitance of the valve and the wiring capacitance. By way of illustration it will be useful here to insert some conventional values in the expressions (5) to (9), i.e.

$R_1=10\,\mathrm{k\Omega}$, $R_2=1\,\mathrm{M\Omega}$, $C_1=10\,\mathrm{pF}$ and $C_k=0.1\,\mu\mathrm{F}$.

We then find:

$R'=9.99\,\mathrm{k\Omega}$, $L'=10^3\,\mathrm{H}$, $C'=10\,\mathrm{pF}$, $\omega_0=10^4\,\mathrm{rad/s}$ and $Q=10^{-3}$.

The value of Q found here is exceptionally small compared with that of conventional resonant circuits (10 to 100). Further, it is already apparent that the circuit in fig. 2 behaves like an LC circuit containing a coil with a very high value of L. Because of the low value of C_1, however, ω_0 is not particularly small. If we make C_1 and C_k much larger, e.g. both 1 μF, then we find from (5) to (9):

$R'=9.98\,\mathrm{k\Omega}$, $L'=10^4\,\mathrm{H}$, $C'=1\,\mu\mathrm{F}$, $\omega_0=10\,\mathrm{rad/s}$ and $Q=9.8\times10^{-2}$.

The resonant frequency is now extremely low, but Q is still very small.

We may now ask whether a higher Q might be obtained by choosing different resistance and capacitance values. This is indeed found to be the case; for Q it is possible to achieve a maximum value of $\frac{1}{2}$, although usually a somewhat lower value is accepted, the reason being that the signals are more strongly attenuated the nearer Q approaches the value $\frac{1}{2}$.

To demonstrate the latter point, we combine (5) and (9) to form the following equation:

$$Q^2+\frac{R'^2}{R_1R_2}=\frac{\dfrac{R_1(C_1+C_k)}{R_2C_k}}{\left\{\dfrac{R_1(C_1+C_k)}{R_2C_k}+1\right\}^2}. \quad (12)$$

The right-hand side has an absolute maximum of $\frac{1}{4}$ for

$$\frac{R_1(C_1+C_k)}{R_2C_k}=1. \quad (13)$$

[3]) The notation V_{g2} is used here because later in the article this voltage will be applied to the grid of another valve (see e.g. fig. 4).

At a very low value of R', or at very high values of R_1 and/or R_2, Q can therefore approach $\frac{1}{2}$. Now R_1 and R_2 cannot be given unlimitedly high values without affecting the operation of the two valves, these resistances being included in the anode and grid circuits respectively. If Q is to approach $\frac{1}{2}$, therefore, R' will have to be small, which means that the signal transmission by the filter will be poor.

Since, moreover, a Q of $\frac{1}{2}$ is still very low, it would not be possible in this way to design a filter that satisfied practical selectivity requirements, if it were not for the fact that the figure of merit can be considerably increased by the use of feedback. This resembles the effect of feedback in an oscillator. Here too, the effective Q of a circuit is increased by feedback until, at an almost infinitely high Q, oscillation occurs.

Circuit using feedback

Feedback can be obtained by applying to the input of the filter a current which is proportional to the output voltage. A suitable circuit for this purpose is shown in *fig. 4*. The output voltage here is obtained by applying the voltage V_{g2} to the grid of a triode B_2 connected as a cathode follower. The feedback occurs through the resistance R_t shunted between the cathodes. The way the circuit works can be understood in simple terms by assuming that the two triodes function as ideal cathode followers. (To approximate to this ideal, the resistances R_{k1} and R_{k2} should be extremely high, and so too should the amplification factors of the valves.) In this case the cathodes carry alternating voltages V_{k1} and V_{k2} which are equal to the respective alternating grid-voltages V_{g1} and V_{g2}. Calculating V_{g2} ($= V_{k2}$) as a function of V_{g1} ($= V_{k1}$), we then find:

$$V_{g2} = -V_{g1}\frac{R_t + R_{k1}}{R_{k1}(R_t - R')}\,\frac{R'}{1 + j\beta\dfrac{R_t}{R_t - R'}Q}\,. \qquad \ldots \quad (14)$$

The anode current of B_1 equals the sum of the currents in R_{k1} and R_t, hence:

$$i_{a1} = \frac{V_{k1}}{R_{k1}} + \frac{V_{k1} - V_{k2}}{R_t}.$$

Using (10) we write:

$$V_{g2} = \frac{-R'}{1 + j\beta Q}\left(\frac{V_{k1}}{R_{k1}} + \frac{V_{k1} - V_{k2}}{R_t}\right),$$

which leads, after some manipulation, to equation (14).

Since the situation is entirely analogous to that in fig. 2, except that eq. (14) now applies instead of eq. (11), we see that the characteristics of the cir-

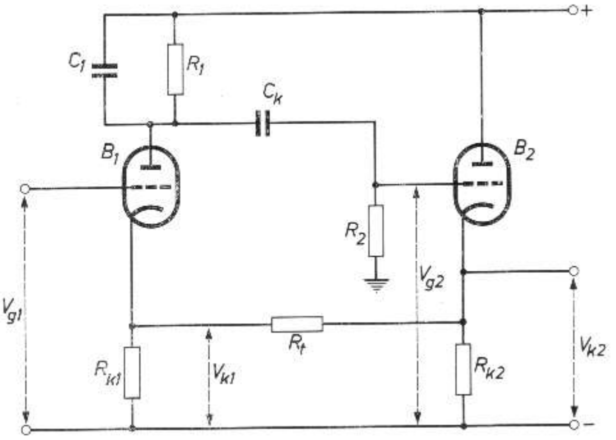

Fig. 4. The figure of merit can be increased by using feedback. In the circuit shown here this is done by introducing the triode B_2 as a cathode follower and a resistance R_t between the cathodes of the two valves.

cuit again correspond to those of an LC circuit, but now with a figure of merit

$$Q' = \frac{R_t}{R_t - R'}Q\,.$$

Thus, by the use of feedback, the figure of merit is increased by a factor $R_t/(R_t - R')$. Raising this factor to 100 or more results in values of Q' comparable with those of LC circuits designed for higher frequencies.

The resonant frequency is not altered by the application of the feedback; the inductance L', however, is increased by a factor $(R_t + R_{k1})/R_t$ and the capacitance C' reduced by the same factor. Instead of the resistance R' from (11) we now have in (14) the resistance $R'(R_t + R_{k1})/(R_t - R')$. The use of feedback thus increases the signal transmission by a factor $(R_t + R_{k1})/(R_t - R')$.

Circuit design; stability

Deciding on the optimum circuit values for a filter under given conditions is usually a complicated problem owing to the large number of variables involved. Moreover the choice of the values of resistances and capacitances is restricted by various practical considerations.

We have seen that the high value of Q required for good selectivity can be achieved by means of feedback. The extent to which this can be done, however, is limited by the fact that a filter is always required to possess a certain *stability*, that is to say its characteristics are required to have a certain constancy under variations that may occur in the components used. In this respect the valves are the greatest danger. If a highly stable circuit is required, they set a limit to the extent to which feedback can be employed.

This can be understood by considering that equation (14), applicable to fig. 4, represents only an approximation since it takes no account of the fact that a cathode follower has an internal resistance which can be said to be roughly equivalent to the reciprocal of the transconductance of the relevant valve. If we take this into account, we find that instead of the resistance R_t shunted between the cathodes of the two valves, we should introduce in the equations a resistance $R_t' = R_t + 1/S_1 + 1/S_2$, where S_1 and S_2 are the transconductances of B_1 and B_2 respectively. Now, for a stable circuit with strong feedback it is most important that the value of R_t' should be highly constant. For if the factor $R_t'/(R_t' - R')$, by which the figure of merit is multiplied, is to be made sufficiently large, R_t' should not differ significantly from R'. A small percentage change in R_t' therefore has a considerable influence on the factor in question, and therefore also on the figure of merit ultimately obtained. Slight variations in the transconductance of the valves can thus cause substantial variations in the Q of the filter.

To minimize this effect it is necessary to use for R_t a highly stable resistance which is large compared with $1/S_1 + 1/S_2$. This means that the circuit should be designed so as to give R' a high value also, and this, according to (5), can only be done by using large values for R_1 and R_2. We have already noted that there are limits to the values we can give to these resistances, and this in fact means a limitation of the degree of feedback that can be adopted when rigorous stability requirements are imposed. In most cases the circuit will therefore be so designed as to obtain the highest possible figure of merit even without feedback, i.e. a Q as close to $\frac{1}{2}$ as a reasonable signal transmission permits. If we choose R_1, R_2, C_1 and C_k so as to satisfy (13), then in accordance with (9) and (5) we have:

$$Q = \tfrac{1}{2}\sqrt{\frac{C_1}{C_1 + C_k}} \quad \text{and} \quad R' = \tfrac{1}{2}R_1 .$$

If, for example, we now take $C_k = \frac{1}{4}C_1$ (to satisfy (13) we must then make $R_2 = 5R_1$), then $Q = 1/\sqrt{5} \approx 0.45$. Since the maximum value of R' is equal to R_1 — i.e. when R_2C_k is large compared with $R_1(C_1 + C_k)$ — the signal transmission at $C_k = \frac{1}{4}C_1$ and $R_2 = 5R_1$ is still 50% of that of the maximum that can be achieved at a given value of R_1. This can be regarded as a reasonable compromise.

In some cases, to ensure that the filter is satisfactorily stable in operation, R_1 and R_2 will have to be higher than is desirable from the point of view of the valves. When the relevant requirements are not too divergent, both can sometimes be met by connecting the anode of B_1 and/or the grid of B_2 to tappings on R_1 and R_2 respectively (see *fig. 5*). It can easily be seen that the filter action is now governed by the total values of R_1 and R_2, while only parts of these resistances are incorporated in the anode circuit of B_1 and the grid circuit of B_2.

Two other limitations of the magnitude of the resistances may be mentioned, the first being the fact that very high resistances possessing high stability (metal film types) are difficult to produce. The other point is that parallel with R_2 is the input capacitance of the valve B_2. If R_2 is high or the frequency for which the relevant filter was designed is not very low (e.g. a few hundred c/s) it may well be that the impedance of the input capacitance of B_2 is not to be disregarded, and since a capacitance of this kind is always a rather unstable quantity, it can have an adverse effect on the stability of the filter. The effect is more pronounced the higher the value of R_2, and

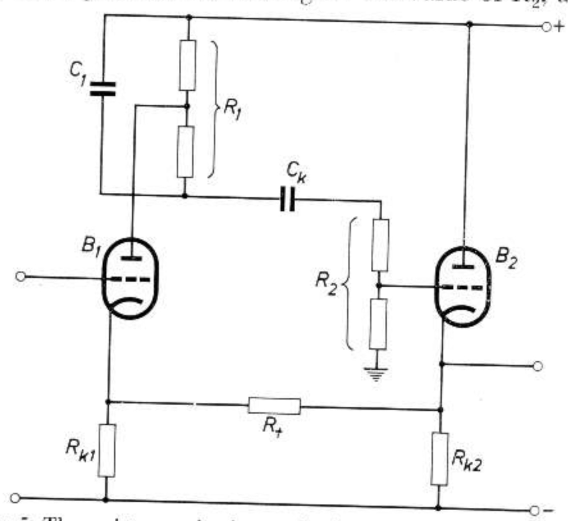

Fig. 5. The resistances in the anode circuit of B_1 and in the grid circuit of B_2 should sometimes be lower than the resistances R_1 and R_2 that govern the filter action. It may then be an improvement to connect the anode of B_1 and the grid of B_2 to tappings on R_1 and R_2.

this too sets a limit to the permissible value of R_2. A means of reducing the influence of the unstable input capacitance of B_2 is to connect a sufficiently large and stable capacitor in parallel with R_2.

In this case the filter proper is formed by two resistors and three capacitors (*fig. 6*). From the equation giving the relation between V_{g2} and i_{a1} we find that for this circuit also we can draw an equivalent circuit as represented in fig. 3. Without going any further into this subject, it will be useful to give the equations which then take the place of (5) to (9):

$$R' = \frac{R_1R_2C_k}{R_1(C_1 + C_k) + R_2(C_2 + C_k)}, \quad \ldots \quad (5')$$

$$L' = R_1R_2C_k, \quad \ldots \quad (6')$$

$$C' = \frac{1}{C_k}(C_1C_2 + C_1C_k + C_2C_k), \quad \ldots \quad (7')$$

$$\omega_0 = \frac{1}{\sqrt{R_1R_2(C_1C_2 + C_1C_k + C_2C_k)}}, \quad \ldots \quad (8')$$

$$Q = \frac{\sqrt{R_1R_2(C_1C_2 + C_1C_k + C_2C_k)}}{R_1(C_1 + C_k) + R_2(C_2 + C_k)}. \quad \ldots \quad (9')$$

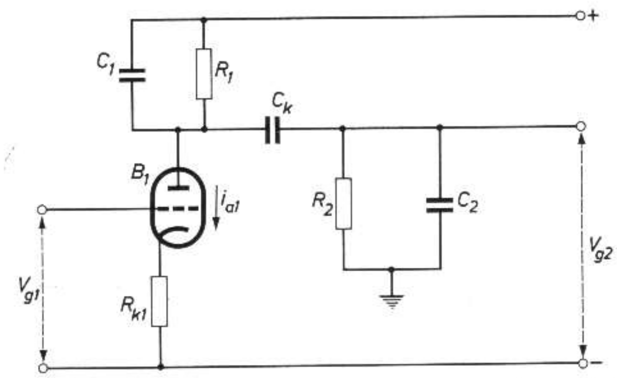

Fig. 6. When the frequency of the signals is not very low and /o the resistance R_2 is very high, the capacitance C_2 may have to be taken into account.

In this case too, as may be deduced from these equations, the maximum value of Q is equal to $\frac{1}{2}$. In this respect, then, the introduction of C_2 makes no difference.

Circuits of high stability

As has been shown, the principal objection to high stability of a filter designed along the lines described above, arises from the variations that may occur in the transconductance of the valves. Substantial improvements can be achieved in this respect by employing circuits in which the feedback is brought about in a different manner. For example, a marked improvement can be obtained by using for the feedback a separate valve, on the principle illustrated in the diagram in *fig. 7*. Valves B_1 and B_2 have the same functions here as in fig. 4. The anode circuit of B_2 now contains a resistance R_{a2}. The alternating anode voltage of B_2 is applied to the grid of B_3, whose anode is connected to the anode of B_1. (The DC biasing of the valves will not be dealt with here.) When the resistances R_{k2} and R_{k3} are very large compared with the reciprocal of the transconductance of the relevant valves, the alternating anode current of B_3 is:

$$i_{a3} = -V_{g2}\frac{R_{a2}}{R_{k2}\,R_{k3}}.$$

This current, together with the alternating current from the anode of B_1, is fed to the input of the actual filter. Here again, then, the strength of the feedback is determined primarily by a number of resistances, i.e. R_{a2}, R_{k2} and R_{k3}. The smaller the reciprocal values of the transconductances of B_2 and B_3 compared with R_{k2} and R_{k3}, the more accurately is this approximation satisfied, and since the latter resistances can readily be made higher than R_t in fig. 4, variations in the transconductance of the valves can easily be made to have less effect on the strength of the feedback in the circuit of fig. 7 than in that of fig. 4; the Q of the circuit in fig. 7 can therefore be more stable.

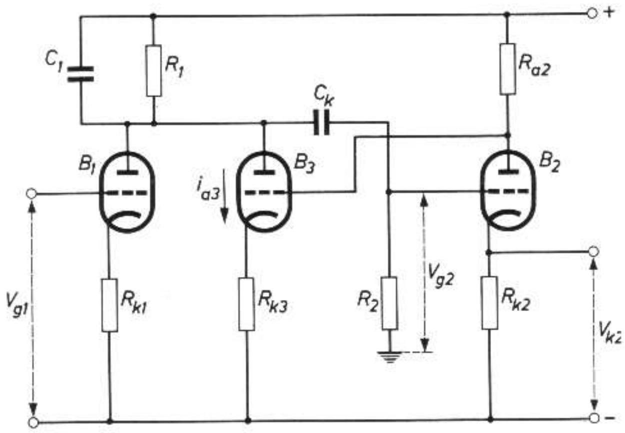

Fig. 7. The stability of the filter can be improved by using a separate valve B_3 for the feedback.

An even greater improvement can be made by using yet another valve, denoted B_4 in *fig. 8*. The operation of this circuit can again be understood by assuming that B_2 functions as an ideal cathode follower, and B_3 likewise. The alternating voltages on both the grid and cathode of B_3 are then equal to $-V_{g2}R_{a2}/R_{k2}$. If we now choose the values of R_2 and R_3 such that $R_2 : R_3 = R_{k2} : R_{a2}$, then the point

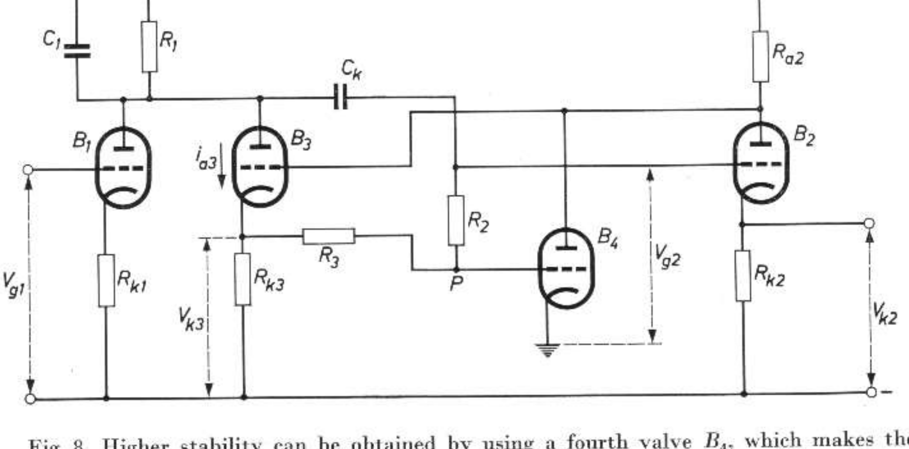

Fig. 8. Higher stability can be obtained by using a fourth valve B_4, which makes the amount of feedback less dependent on variations in the transconductances of B_2 and B_3.

P will carry no alternating voltage with respect to earth, and therefore no alternating anode-current will flow in B_4. The operation of the circuit in fig. 8 is then identical with that in fig. 7. (The only difference is that fig. 7 contains the resistor R_{k3} for alternating current in the cathode lead of B_3, whereas fig. 8 contains R_{k3} and R_3 in parallel.) Any change in this situation produces an alternating voltage at point P. This gives rise in B_4 to an alternating current which has a corrective effect, owing to the anode of B_4 being connected with that of B_2. In this way, then, the voltages and currents always adjust themselves so that there is virtually no alternating voltage at point P, and it can easily be seen that in this case the ratio of the alternating current i_{a3} and the alternating voltage V_{g2} is governed solely by the resistances R_2, R_3 and R_{k3}. Owing to the corrective effect of B_4, variations in the transconductance of the valves have no effect on this ratio, and therefore the feedback is extremely stable, making it possible to give the figure of merit of the filter an exceptionally high and stable value.

Obtaining a flatter frequency characteristic

As we have shown, the frequency characteristic of a filter as drawn in figs. 4 to 8 corresponds to that of an LC circuit. In many cases a characteristic will be required that is flatter at the top and has steeper sides, as can be obtained for example with the aid of two or more non-coupled LC circuits whose resonant frequencies do not coincide (staggered tuning). A frequency characteristic having an exceptionally flat and broad top can be obtained by this method, using three tuned circuits, the figure of merit of one of them being half that of the two others, and the latter two being tuned to specific frequencies which are higher and lower than the resonant frequency of the first one. This principle can also be applied by using, instead of LC circuits, active filters on the principle described above; the alignment of the networks, however, is rather laborious as it is with passive filters.

Another method of obtaining a flat frequency characteristic with passive LC circuits is to couple circuits which are all tuned to the same frequency. This principle can also be applied to the active filters described here, the "coupling" being achieved by introducing negative feedback between pairs of filters.

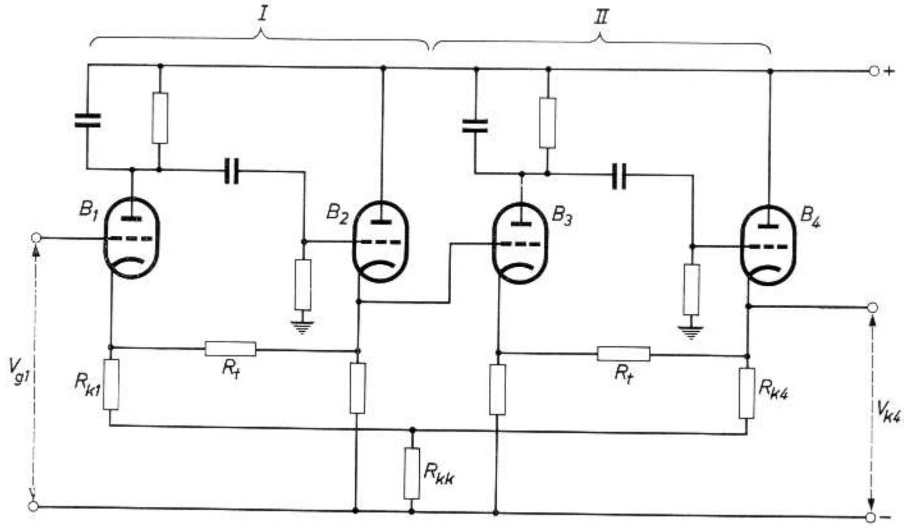

Fig. 9. Combination of two active filters, I and II, with negative feedback obtained by means of R_{kk}. The frequency characteristic achieved with this circuit corresponds to that of two coupled LC circuits.

An example of such a circuit is shown in *fig. 9*. The two filters are denoted by I and II. The negative feedback takes place via a common resistance R_{kk} introduced in the cathode lead of the first valve (B_1) in the first filter and of the second valve (B_4) in the second filter [4]).

Making some simplifying assumptions, i.e. by treating B_2 and B_4 as ideal cathode followers and assuming that R_{kk} is small compared with R_{k1} and R_{k4}, it is

[4]) Instead of the resistances R_{k1}, R_{k4} and R_{kk}, a delta connection of resistances might have been used. A drawback, however, is the very high value that would then be needed for the feedback resistor, which would be difficult to make with sufficient stability. In this respect the small resistance R_{kk} presents no difficulties.

easily computed that the relation between the output voltage V_{k4} and the input voltage V_{g1} is given by the expression:

$$V_{k4} = \frac{AV_{g1}}{(1+j\beta Q') + Ap} . \quad . \quad . \quad (15)$$

Here A is the gain of the circuit without negative feedback at the resonant frequency, i.e. at $R_{kk} = 0$, and $\beta = 0$, while

$$p = \frac{R_{kk}}{R_{k4}} \; \frac{R_t}{R_{k1} + R_t} .$$

This quantity is thus a measure of the strength of the feedback. If we now take $Ap = 1$, say, then (15) becomes:

$$V_{k4} = \frac{AV_{g1}}{(1+j\beta Q')^2 + 1} , \quad . \quad . \quad . \quad (16)$$

and the relation between the moduli of V_{k4} and V_{g1} is then given by

$$|V_{k4}| = \frac{A}{\sqrt{4 + \beta^4 Q'^4}} \; |V_{g1}| . \quad . \quad (17)$$

The behaviour of $|V_{k4}|/|V_{g1}|$ as a function of β is in this case identical with that in a band-pass filter consisting of two critically coupled LC circuits. The characteristic is flatter and has steeper sides than that belonging to a single LC circuit.

By using combinations of such filters with different "couplings" (Ap) it is possible to meet widely varying requirements with regard to the frequency characteristic. A filter as in fig. 4 can, for example, be connected in cascade with a filter as in fig. 9; with suitable dimensioning, a frequency characteristic can then be obtained corresponding to that of three LC circuits with staggered tuning. The conformity of the equations makes it possible to treat the design of such configurations mathematically in the same way as the design of filters using LC circuits.

Filters I and II in fig. 9 are circuited on the principle represented in fig. 4. Obviously, to achieve greater stability one can also use the circuits of fig. 7 or fig. 8 as "building bricks".

Low-pass filters

The filters described above are of the band-pass type, that is to say the frequency band which is transmitted by the filter is limited on both the low and high frequency sides. For certain experiments, however, *low-pass* filters are wanted, i.e. filters which pass all signals components with frequencies below a certain critical frequency. A filter of this type is approximated in the first instance by a single resistor and a single capacitor (see *fig. 10*). One can think of a low-pass filter as being derived from the band-pass type by making the capacitance C_k infinitely large, i.e. by short-circuiting it, thus making the resonant frequency zero. (The resistors R_1 and R_2 in fig. 2 are now in parallel for alternating current and in fig. 10 are replaced by a single resistance R.)

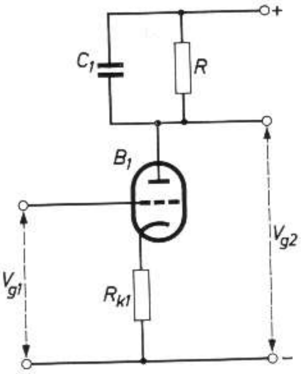

Fig. 10. AC circuit for a low-pass filter.

Introducing feedback here in the same way as represented in fig. 4, we obtain the diagram shown in *fig. 11*. The relation between input and output signal is now:

$$V_{k2} = V_{g2} = -V_{g1} \frac{R_t + R_{k1}}{R_t - R} \; \frac{R}{R_{k1}\left(1 + j\omega \dfrac{R_t}{R_t - R} C_1 R\right)} . \quad . \quad . \quad . \quad (18)$$

It can be seen from this that the feedback has the same effect on the frequency characteristic as increasing C_1 by a factor $R_t/(R_t - R)$. For limiting the pass-band to very low frequencies it will be easier to use a high value of C_1 than to introduce feedback, and the latter will therefore seldom be used for a simple low-pass filter.

The situation is different, however, if here too use is made of the possibility of connecting two stages in cascade and introducing negative feedback. An

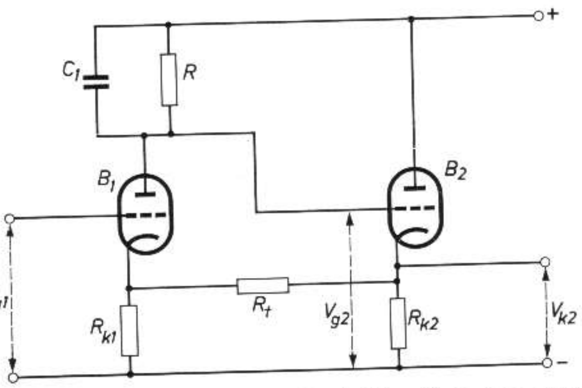

Fig. 11. Low-pass filter with feedback. The effect of the feedback on the frequency characteristic is the same as that of an increase in the capacitance C_1.

arrangement of this kind is shown in *fig. 12*, using two identical filters and negative feedback via the resistor R_{tk} [5]). Assuming again that the resistances in the cathode leads are very high, we can derive the following expression for the relation between the output voltage V_{k3} and the input voltage V_{g1}:

$$V_{k3} = \frac{A(1+p)V_{g1}}{(1+j\omega C_1 R)^2 + Ap} . \quad . \quad . \quad (19)$$

Here $A = R^2/(R_{k1}R_{k2})$ and $p = R_{k1}/R_{tk}$. Since we have assumed that R_{k1}, R_{k2} and R_{k3} are very large,

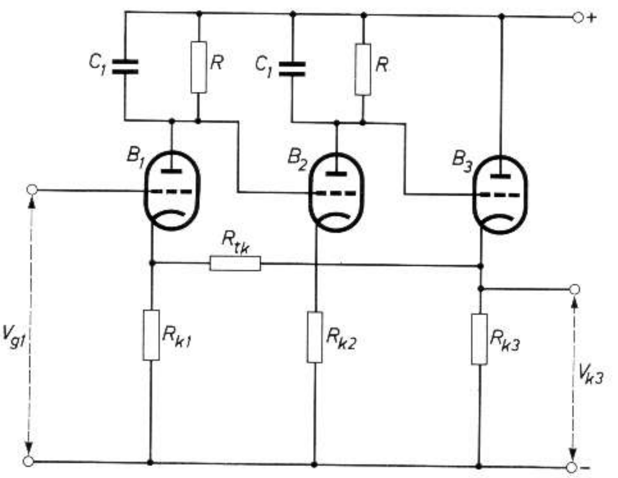

Fig. 12. Low-pass filter consisting of the cascade arrangement of two identical filters as in fig.10, negative feedback being introduced by means of the resistance R_{tk}.

A is the gain of the circuit at zero frequency and with the negative feedback out of operation (i.e. at $R_{tk} = \infty$). Further, p is a measure of the strength of the negative feedback.

From (19) it follows that for $\omega = 0$ the output voltage is:

$$V_{k3\cdot 0} = \frac{A(1+p)V_{g1}}{1+Ap} . \quad . \quad . \quad . \quad (20)$$

In fig. 13 the ratio

$$\frac{|V_{k3}|}{|V_{k3\cdot 0}|} = \left| \frac{1+Ap}{(1+j\omega C_1 R)^2 + Ap} \right| \quad . \quad (21)$$

is plotted as a function of ω for various values of Ap,

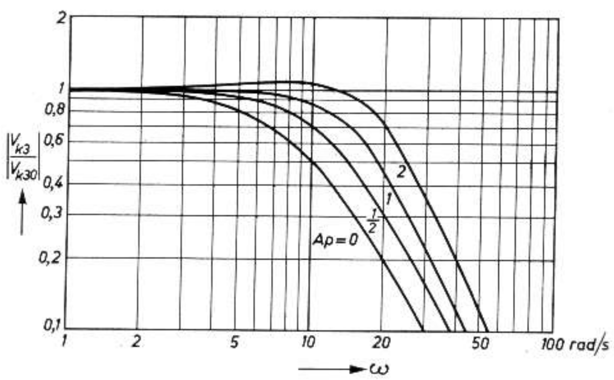

Fig. 13. Frequency characteristics for a low-pass filter as in fig. 12.

keeping the product C_1R constant for all curves (0.1 s). The curve drawn for $Ap = 0$ applies to the case without negative feedback. Comparison with the other curves shows that the use of feedback can lead to a much flatter frequency characteristic, which cannot be obtained with passive *RC* circuits.

With low-pass filters too, it is possible to build more extensive networks by combining filters having different C_1R values. By suitable dimensioning, curves can be obtained which are even flatter at the top and have a steeper descending portion.

Application as a delay network

The analogy existing between *LC* circuits and the active filters described above makes it possible to use these active filters in cases where it is required to delay signals of extremely low frequency without causing any distortion. This can be useful, for example, when building an electrical analogue of a control system involving transmission lags. An electrical analogue of this type can be used for analyzing the stability conditions of the control system.

Where a network is required to transmit each signal undistorted but with a certain retardation, the phase delay must be independent of the frequency, and consequently the phase shift suffered by the various components must be proportional to their frequencies. Furthermore, the amplitude ratio between input and output must be independent of the frequency.

A familiar circuit with which this can be realized in a limited phase-shift region is shown in *fig. 14a*. A less familiar circuit with which the same results can be achieved, and which offers advantages in connection with the following considerations, is given in fig. 14*b*. Assuming for simplicity that R_1 and R_2 are large with respect to R and R_k, and choosing the resistances so as to satisfy the equation:

$$\frac{R_1}{R_2} = \frac{2R_k}{R}, \quad . \quad . \quad . \quad . \quad . \quad (22)$$

[5]) Since neither of the two individual filters is provided with feedback, this circuit can be designed with only three valves, whereas in fig. 9 four valves were needed. Furthermore, unlike fig. 9, the negative feedback here is obtained with a delta connection of the resistances R_{k1}, R_{k3} and R_{tk}. The reason for this is that, owing to the absence of feedback in the individual filters, the total gain in fig. 12 is much smaller than in fig. 9. To obtain sufficient effect from the feedback, a common resistance in series with R_{k1} and R_{k3} would therefore have to be much larger than R_{kk} in fig. 9. In this case the introduction of a resistance between the cathodes of B_1 and B_3 is a better solution, because such a resistance has less influence on the DC biasing of the two valves.

then the relation between V_g and V_u is given by:

$$\frac{V_g}{V_u} = \frac{R_1 + R_2}{R_2} \frac{1 + j\omega C_1 R}{-1 + j\omega C_1 R}. \quad (23)$$

The modulus of this expression is indeed independent of ω, and the argument is $2 \tan^{-1}\omega C_1 R \pm 180°$. The voltage V_u thus lags in phase behind V_g by an angle $2 \tan^{-1}\omega C_1 R \pm 180°$. We shall henceforth consider only the frequency-dependent term $\varphi = 2 \tan^{-1} \omega C_1 R$ of this phase shift, since the constant quantity $\pm 180°$ can be compensated, e.g. by using two stages in cascade. In *fig. 15* curve *a* gives the variation of φ as a function of ω. At small values of $\omega C_1 R$, then, $\tan^{-1}\omega C_1 R \approx \omega C_1 R$, and therefore φ is approximately proportional to ω. This applies, however, only up to a limited value of φ (e.g. 60°). The proportionality region can be extended by connecting in cascade with a circuit as in fig. 14 one or more elements whose phase characteristics have linear portions at frequency values differing from zero. This can be done with a circuit as shown in *fig. 16*, for example, which differs from that in fig. 14*b* in that the anode circuit of the valve contains an *LC* circuit instead of the *CR* combination. In this configuration the following relation exists between V_g and V_u:

$$\frac{V_g}{V_u} = \frac{R_1 + R_2}{R_2} \frac{1 + j\beta Q}{-1 + j\beta Q}. \quad (24)$$

(Here β again represents $\omega/\omega_0 - \omega_0/\omega$.) The voltage V_u now lags in phase behind V_g by an angle $2 \tan^{-1} \beta Q \pm 180°$. Fig. 15 gives a plot of $\varphi = 2 \tan^{-1} \beta Q + 180°$ as a function of ω (curve *b*). Using circuits as in fig. 14 and fig. 16 connected in cascade, a phase shift is obtained between input and output voltage which can be found by adding the ordinates of curves *a* and *b* (curve *c*). Choosing suitable circuit values we can now obtain a linear variation over a much wider region than is possible using a circuit as in fig. 14 alone.

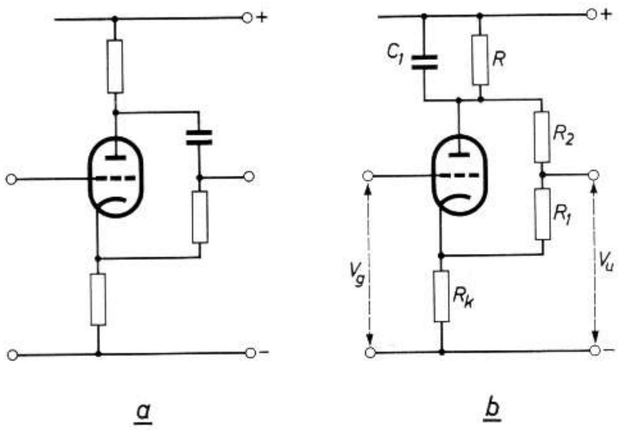

Fig. 14. Circuits in which the signals have a constant phase lag within a limited frequency range, and a constant ratio between the amplitudes of input and output signal.

Fig. 15. Phase characteristic of the circuits in fig. 14 (*a*) and of the circuits in figs. 16 and 17 (*b*). Connected in cascade and suitably dimensioned, these circuits give the phase characteristic *c*.

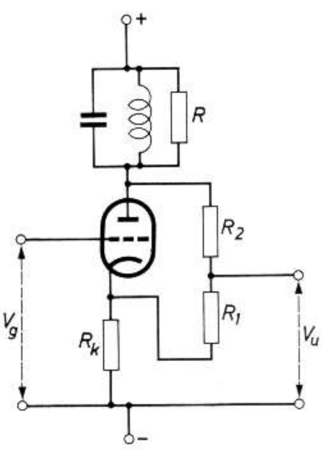

Fig. 16. Circuit giving a phase characteristic that has a linear portion round a frequency differing from the value zero.

The principle described can also be applied at very low frequencies when a filter as in fig. 4 is to be used instead of an *LC* circuit. The voltage divider between input and output can be produced by using two resistors R_{t1} and R_{t2} instead of the feedback resistor R_t in the circuit shown in *fig. 17*. The ratio of these resistances should now satisfy the expression:

$$\frac{R_{t1}}{R_{t2}} = \frac{2R_{k1}}{R'}, \quad (25)$$

where R' is given by (5). By means of a cascade arrangement of the circuits of fig. 14 and fig. 17, we can thus obtain a phase characteristic of the form represented by curve *c* in fig. 15.

The "proportionality region" can be still further extended by adding to the circuit one or more filters as in fig. 17, having different resonant frequencies. If these resonant frequencies and the figures of merit are properly chosen, a phase characteristic can be obtained which is almost linear in a wide range of frequencies from zero upwards. This means that signals that possess components lying only in this region have a constant phase delay and therefore undergo a certain delay without any distortion. The figures of merit of the various circuits connected in cascade should, to a good approximation, be virtually proportional to the resonant frequencies. This implies that some of these filters require a fairly high Q, which, within certain limits, can be achieved by means of feedback.

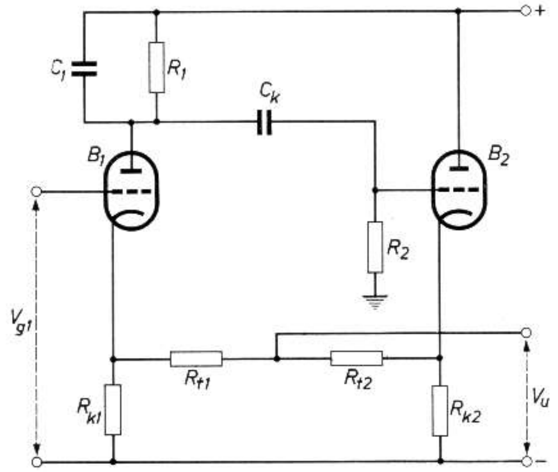

Fig. 17. Circuit which, suitably dimensioned, can be made identical at low frequencies with a circuit as shown in fig. 16.

Final considerations

In the foregoing we have referred to the analogy existing between certain filters composed of resistances and capacitances, and filters using one or more *LC* circuits. In conclusion we should point to a *difference* between these two circuits that can be of importance in practical applications, namely the fact that the feedback employed to achieve a reasonably high figure of merit causes an increase in the *noise level*. Because of this fact, filters based on the principle described in this article can only be used when the signal level is sufficiently high, higher than that at which filters with *LC* circuits can be employed.

Finally, it may be mentioned that circuits on the principles described here can also be designed with transistors. In this case, however, the input resistance is much smaller than with valves, which restricts the choice of the circuit elements that can be used.

Summary. Certain active electric networks composed of amplifying valves, resistors and capacitors have characteristics that correspond to those of passive networks containing inductors. At very low frequencies, where coils have certain disadvantages, good use can be made of active networks of this kind. The authors discuss the design of band-pass and low-pass filters, using feedback to achieve a reasonably high figure of merit. They also deal with various circuits for giving these filters exceptionally high stability.

By connecting two filters in cascade and using negative feedback, a frequency characteristic corresponding to that of two coupled *LC* circuits can be achieved. Finally, reference is made to the possibility of designing networks in this way whose amplitude characteristic is flat within a given frequency range and whose phase characteristic is practically linear, so that signals only containing components in this frequency range can pass through these networks with a certain delay without suffering distortion.

PRODUCTION CENTRE FOR LIQUID NITROGEN

In the last ten years many laboratories have been equipped with a Philips gas refrigerating machine, enabling them to supply their own liquid air requirements. Since in many cases there is a preference for pure liquid nitrogen, many of these machines are used in combination with an air fractionating column, also developed by Philips, which is adapted to the gas refrigerating machine. In our own laboratories — as elsewhere — it is found that the ease with which the liquid air or nitrogen can be obtained greatly stimulates its use. The small installation earlier described [1] [2]), which in its present form delivers 6.5 litres of liquid nitrogen an hour, often turns out after some time to be too small. With this in mind, a gas refrigerating machine has been developed which combines four of the original single-cylinder machines. A larger air fractionating column adapted to the large machine is also available.

The photograph on p. 341 was taken in the liquid air

1) J. W. L. Köhler and C. O. Jonkers, Fundamentals of the gas refrigerating machine, and Construction of a gas refrigerating machine, Philips tech. Rev. **16**, 69-78 and 105-115, 1954/55.

2) J. van der Ster and J. W. L. Köhler, A small air fractionating column used with a gas refrigerating machine for producing liquid nitrogen, Philips tech. Rev. **20**, 177-187, 1958/59.