to avoid this the part of the instrument round the lenses has been boxed in and a small silica gel dryer placed inside. If condensed water vapour is allowed to remain on a bloomed lens for more than a few hours it leaves a permanent mark and the phase of the beam passing through is slightly modified.

5. PHOTOGRAPHIC TECHNIQUE

The time taken to punch masks and place them in the diffractometer is so short that, in obtaining optical transforms, the photographic processing is the main factor in determining the total time involved. It has, therefore, become important to cut down the processing time to a minimum; this has been achieved by the use of modern rapid developers and fixers. The requirements for the photographic work are small grain size, reasonable contrast, short exposure and development time, combined with control of the processing.

Until recently Leitz two-bath developer[12] has been used in conjunction with an ammonium thiosulphate fixing bath. Velox developer or D8 has been used where greater contrast is required. A series of tests has recently been carried out on various types of films and developers, the results of which are given in Table 5.

ACKNOWLEDGEMENTS

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Table 5. *Comparison of various combinations of films and processes*

	Test 1	*Test 2*	*Test 3*	*Test 4*
Film	(Kodak) Pan Microfile	Pan Microfile	Pan Microfile	Quick finish
Developer	Leitz two-bath	Velox	Quick finish	Quick finish
Fixer	Ammonium thiosulphate	Ammonium thiosulphate	Quick finish	Quick finish
Stop bath	2% acetic	2% acetic	None needed	None needed
Total time	11 min	3 min	50 s	50 s
Grain size	Small	Fairly small	Small	Too large
Control	Good	Good	Fairly good	Fairly good
Contrast	Good gradation	Higher than 1	As 1	Fair
Sensitivity	1 and 2 are about equal; 3 and 4 give twice the sensitivity of 1 and 2			

From the table it will be seen that the most useful combination is given by test 3. This cuts down the processing time for the negative to 50 s. Prints may be made directly before washing. It was found that a development time of 20 s at 20° C and a fixing time of 30 s at the same temperature were suitable. No significant change in the density of the negative was recorded for an error of ± 2 s in the development time. Leitz two-bath and D8 developers still have their advantages for special purposes.

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A low frequency random signal generator

By J. C. WEST, Ph.D., D.Sc., A.M.I.E.E., Electrical Engineering Department, University of Manchester, and G. T. ROBERTS, B.Sc., Bruce Peebles and Co. Ltd., Edinburgh

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Conventional random signal sources such as photo-multipliers, gas discharge tubes, etc., although good at medium and high frequencies are unreliable at frequencies below about 50 c/s, due to flicker effect. The paper describes a simple method of generating a low frequency random signal having a spectrum extending down to zero frequency, and using a gas discharge tube as the primary noise source. The gas discharge tube has been chosen because of the relatively large noise power available, for instance, this may be 10 V r.m.s. in the band spectrum up to 10^7 c/s. The probability distribution of the resulting signal approaches a Gaussian form. The difference from a true Gaussian is very small and may be neglected in most experimental work.

INTRODUCTION

A gas discharge tube will provide Gaussian noise having a flat spectrum extending down to about 50 c/s. A portion of this spectrum in the high audio frequency range is selected by a narrow band filter with central frequency f_0. The output of the filter has the appearance of a sine wave of frequency f_0 but with randomly varying amplitude, the frequency of the fluctuations being of the order of the bandwidth of the filter. Peak rectification of this signal yields a low frequency waveform with a spectrum extending down to zero frequency.

Since the output of a peak rectifier can never reverse polarity, the low frequency waveform will have a d.c. component, and its instantaneous amplitude will always be greater than zero. The actual amplitude distribution is found to be Rayleighan. If two statistically independent signals of this type are added together such that their d.c. components cancel, the result is a random signal having a symmetrical distribution about zero amplitude, and this approaches the Gaussian distribution very closely. The spectrum of the combined signal again extends down to zero frequency, the upper cut-off being determined by the bandwidth of the band-pass filters. Fig. 1

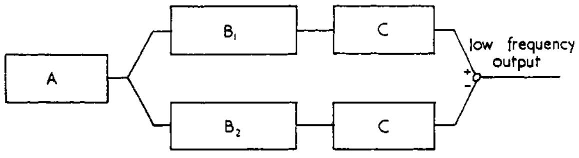

Fig. 1. Schematic diagram of low frequency noise generator

A, wide band Gaussian noise source; B_1, narrow band filter centre frequency f_1; B_2, narrow band filter centre frequency f_2; C, peak rectifier with smoothing.

is a schematic diagram of the complete system. It is permissible to use a single noise source since the outputs of the two filters will be statistically independent if their pass bands are sufficiently separated.

implies that the signal $x(t)$ may be considered as a carrier wave of frequency ω_0 modulated by a slowly varying function of time $R(t)$. From equations (5) we see that $y_1(t)$ and $y_2(t)$ have similar amplitude spectra which are identical with the original band-pass filter spectrum but with a shift of the frequency origin to ω_0, so that the mid-band response of the filter now represents the amplitude of the zero frequency components in y_1 and y_2.

By the central limit theorem y_1 and y_2 must approach normal amplitude distributions since each is the sum of a large number of independent random variables. Demodulation of $x(t)$ by multiplication by a signal $2 \cos \omega_0 t$ yields the low frequency component $y_1(t)$ plus higher frequency components which can be filtered off. This would be a convenient method for the generation of low frequency Gaussian signals,[2] but it suffers from certain practical disadvantages. Firstly, it is necessary to ensure an extremely high degree of frequency stability in both the tuned filter and the local oscillator supplying the demodulating signal, since any frequency drift would introduce unpredictable and intolerable fluctuations in the output signal. Secondly, the demodulator, if of the electronic type, is liable to introduce d.c. drift, whilst mechanical chopper type of demodulators limit the highest carrier frequency which can be used. To overcome these disadvantages the present system employs a simple peak detector. This is found to introduce negligible d.c. drift, but unfortunately the amplitude distribution is no longer normal.

BANDWIDTH AND PROBABILITY DISTRIBUTION

A random noise signal $x(t)$ may be represented[1] by the sum of a large number of sinusoidal components with random phase relationships, thus

$$x(t) = \sum_{n=1}^{N} c_n \cos (\omega_n t + \phi_n) \tag{1}$$

the ϕ_n's being uniformly distributed in the range 0–2π. If $x(t)$ is the output of a filter subject to a wide band white noise input, then the c_n's of equation (1) are determined solely by the frequency characteristic of the filter as follows,

$$c_n = |G(\omega_n)| \Delta\omega \tag{2}$$

where $G(\omega)$ is the transfer function of the filter, and $\Delta\omega$ is the increment in frequency between the components of the series.[1]

Equation (1) may be written,

$$x(t) = \sum_{n=1}^{N} c_n \cos (\omega_n t - \omega_0 t + \phi_n + \omega_0 t)$$

$$= y_1 \cos \omega_0 t - y_2 \sin \omega_0 t \tag{3}$$

$$= R \cos (\omega_0 t + \theta) \tag{4}$$

where
$$\left.\begin{aligned} y_1(t) &= \sum_{n=1}^{N} c_n \cos [(\omega_n - \omega_0)t + \phi_n] \\ y_2(t) &= \sum_{n=1}^{N} c_n \sin [(\omega_n - \omega_0)t + \phi_n] \end{aligned}\right\} \tag{5}$$

and
$$\left.\begin{aligned} R(t) &= \sqrt{(y_1^2 + y_2^2)} \\ \theta(t) &= \tan^{-1} \frac{y_2}{y_1} \end{aligned}\right\} \tag{6}$$

If $G(\omega)$ is the transfer function of a fairly narrow band filter, and ω_0 represents the mid-band frequency, equation (4)

AMPLITUDE DISTRIBUTION FOR $R(t)$

As pointed out in the previous section, the amplitude distributions of both y_1 and y_2 must approach the Gaussian form with the same standard deviation σ.

$$p(y_1) = p(y_2) = p(y) = \frac{1}{\sqrt{(2\pi)}\sigma} e^{-y^2/2\sigma^2} \tag{7}$$

where
$$\sigma^2 = \Sigma \tfrac{1}{2} c_n^2 = \overline{x^2} \tag{8}$$

Also, since $\overline{y_1 y_2} = 0$, the joint probability of y_1 and y_2 is simply the product of their separate probabilities. The probability that the signal $R = \sqrt{(y_1^2 + y_2^2)}$ lies within the elementary rectangle y_1, $y_1 + dy_1$, y_2, and $y_2 + dy_2$ may be written,

$$P(y_1 y_2) = \frac{1}{2\pi\sigma^2} e^{-(y_1^2 + y_2^2)/2\sigma^2} dy_1 dy_2$$

Changing from rectangular to polar co-ordinates we have from equation (6),

$$y_1^2 = y_2^2 = R^2$$

$$dy_1 dy_2 = R dR d\theta$$

$$P(y_1 y_2) = P(R\theta) = \frac{1}{2\pi\sigma^2} e^{-R^2/2\sigma^2} R dR d\theta$$

Integrating this expression over all values of θ from 0–2π we arrive at the probability distribution with respect to R alone.

$$\left.\begin{aligned} p(R) &= \frac{1}{\sigma^2} R e^{-R^2/2\sigma^2}, \quad R > 0 \\ &= 0 \qquad\qquad\quad R < 0 \end{aligned}\right\} \tag{9}$$

This is the Rayleigh distribution* and is plotted in Fig. 2.

* This distribution is also known as Circular Gaussian.

$R(t)$ is seen to have a finite probability of reaching very large amplitudes, but it can never cross the zero amplitude level. By combining two signals of this type such that their d.c.

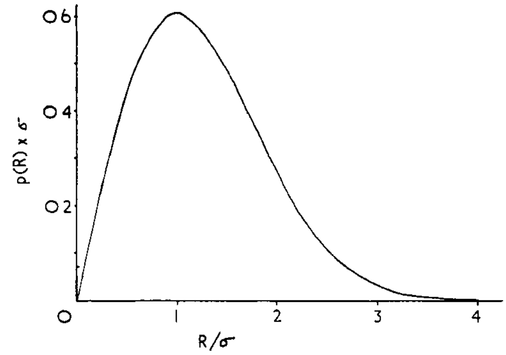

Fig. 2. The Rayleigh probability distribution

$$p(R) = \frac{1}{\sigma^2} Re^{-R^2/2\sigma^2}$$

components cancel, a symmetrical distribution is obtained which is almost normal.

DISTRIBUTION OF THE OUTPUT SIGNAL

The theoretical distribution obtained by combining two Rayleigh distributions cannot be expressed explicity in terms of the signal amplitude. The distribution can be written thus,

$$p(z) = \frac{e^{-a}}{\sqrt{(2\psi)}}\left\{\Gamma_\infty(\tfrac{3}{2})\left[1 - \frac{\Gamma_a(\frac{3}{2})}{\Gamma_\infty(\frac{3}{2})}\right] - a\Gamma_\infty(\tfrac{1}{2})\left[1 - \frac{\Gamma_a(\frac{1}{2})}{\Gamma_\infty(\frac{1}{2})}\right]\right\}$$

where $$a = \frac{z^2}{2\psi}$$

and $$\Gamma_a(n) = \int_0^a t^{n-1}e^{-t}dt.$$

The function $\Gamma_a(n)$ is an incomplete gamma function and has been tabulated by Pearson.[4] The distribution $p(z)$ is plotted in Fig. 3 together with a normal distribution, each curve being normalized with respect to its r.m.s. value. The departure from the ideal distribution is seen to be so small that it may be neglected entirely in most experimental work.

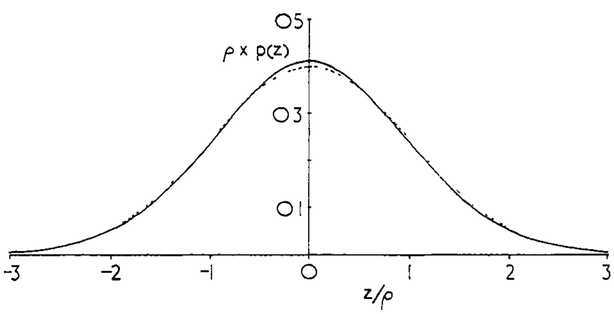

Fig. 3. Theoretical distribution for z together with the normal curve

- - - - - normal distribution $\frac{1}{\sqrt{(2\pi)}\rho}e^{-z^2/2\rho^2}$

——— theoretical distribution of z

THE PRACTICAL SYSTEM

The primary noise source is an argon filled thyratron operating in a magnetic field[3] with heater and h.t. voltages

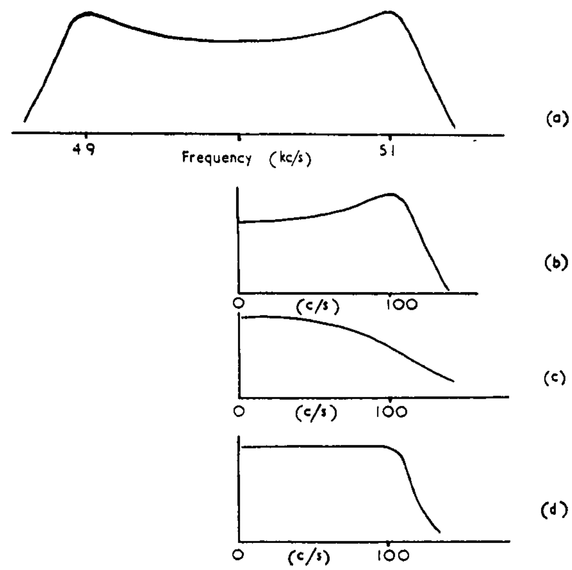

Fig. 4. Filter response and output noise spectrum

(a), filter pass band characteristic; (b) spectrum of signal at output of rectifier; (c) response of low pass R-C filter and (d) overall response at output of noise generator.

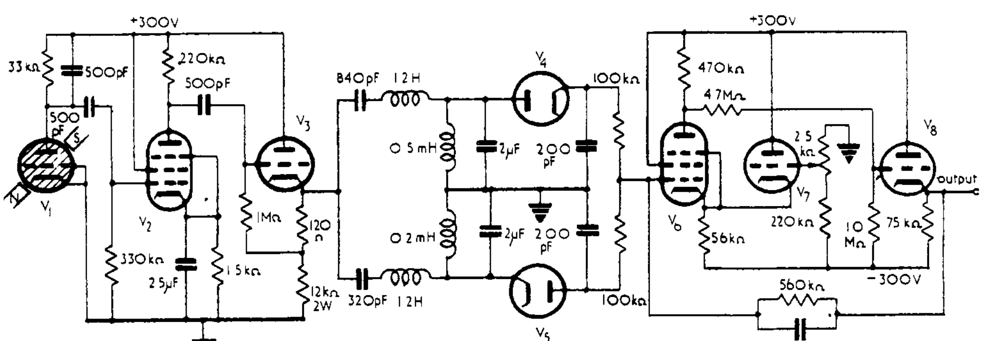

Fig. 5. Circuit diagram of low frequency noise generator

V_1, 6D4, V_2, 6AM6, V_3, 2 × ½ 12AT7; V_4, V_5, 6AL5; V_6, 6AM6; V_7, V_8, 12AT7.

stabilized in order to minimize fluctuations in noise power. The band-pass filters are simple half-sections centred at 5 kc/s and 8 kc/s. The overall bandwidth of each filter is of the order of 250 c/s. The frequency characteristics are not flat topped but rise by about 3 dB at each edge. This gives a low frequency noise signal which rises to a peak at about 100 c/s. The rectifiers are followed by a low frequency R-C filter which cuts off at about 100 c/s. This causes the overall frequency characteristics of the noise to be flat up to 100 c/s and then fall away at higher frequencies. The details of the filters are shown in Fig. 4. It is seen that the spectrum of the output signal is fairly well defined and cuts off rapidly above 100 c/s. This characteristic is obtained without elaborate low-pass filters which usually require unrealistic values of inductance at low frequencies. The complete circuit diagram is given in Fig. 5. This again demonstrates the simplicity of the system.

CONCLUSION

A simple random signal generator is described and shown to give a signal spectrum extending down to zero frequency, and an amplitude distribution approaching the normal curve sufficiently close to be of use in a wide variety of experimental applications. The device was originally developed for use with an analogue computer, and provides a continuous source of random disturbances for statistical investigation of the behaviour of dynamic systems.

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The time behaviour of logarithmic amplifier input circuits

By T. P. FLANAGAN, M.Sc., A.M.I.E.E., A.Inst.P.,* Marconi Instruments Ltd., St. Albans, Herts.

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The non-linear nature of the logarithmic amplifier input circuit causes the time constant to vary with current, and three definitions of effective time constant are examined, based on an analysis of the time behaviour of the input circuit to a step change of input current.

In nuclear reactor instrumentation systems it is usual practice to measure neutron flux by means of a logarithmic amplifier, which measures the current from an ionization chamber acting as a neutron flux detector. The logarithmic characteristic is desirable for two reasons: (*a*) the output of the logarithmic amplifier, when the input is rising exponentially with time, can be differentiated to give a measure of the reactor doubling time, and (*b*) wide range of flux level can be presented on a single meter or recorder scale.

The logarithmic characteristic is obtained by using the exponential voltage-current relationship of a diode operating in the retarding field region,[1] or the grid voltage-grid current relationship of a pentode valve,[2] using the screen grid of the pentode to maintain automatically the anode current at a constant value. The following discussion uses the diode as a model, although the argument will apply equally well to the pentode circuit.

The principle of the method is shown in Fig. 1, which gives the equivalent input circuit of the logarithmic amplifier. The voltage v across the diode is given in terms of the diode current i by

$$i = i_0 \exp (ev/k\theta) \quad (1)$$

where i_0 is a constant characteristic of the diode, e is the electronic charge, k is Boltzmann's constant and θ is the cathode temperature of the diode. The diode current i is derived from the ionization chamber which is a constant current device. Therefore the grid of the amplifier assumes a voltage v given by equation (1).

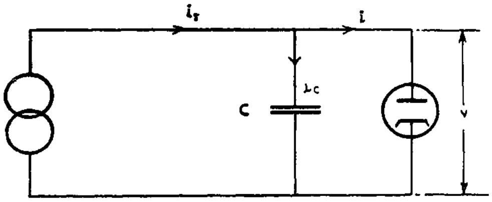

Fig. 1. Equivalent input circuit of logarithmic amplifier

In any practical system the constant current source of the ionization chamber is shunted by capacitance. This is due to the self-capacitance of the chamber itself and the capacitance of the cable carrying the signal from the chamber to the logarithmic amplifier. Thus there will be a time delay in the response of the amplifier to changes in signal current. However, because of the non-linear relationship of input current and voltage this time delay cannot be assessed in terms of the time constant associated with a capacitance-resistance circuit, since there exists no constant resistance across the capacitance. It is possible, of course, to define an instantaneous time constant as the product of the capacitance and the differential resistance dv/di at a particular value of current. From equation (1), by differentiation we have

$$R = dv/di = k\theta/ei \quad (2)$$

so that the time constant T is given by

$$T = Ck\theta/ei \quad (3)$$

Although this is satisfactory as a definition of the time constant at a given value of current, it is of little use when

* Now at the British Scientific Instrument Research Association, Chislehurst, Kent.