A Method of Locking Oscillators in Integral and Non-Integral Frequency Ratios

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The output voltages of two audio oscillators required to operate in an integral or non-integral frequency ratio are applied to a non-linear mixer. The appropriate zero beat is used to provide a suitable correction voltage to a frequency determining circuit forming part of one of the oscillators. A practical arrangement is described by which an *R-C* oscillator may be locked with a 1 000 c/s maintained fork in quite high orders of frequency ratio (e.g. 17 : 7). Further development of the principle is discussed.

Since Appleton[1] in 1922 first gave the matter careful theoretical study, the problem of controlling or "locking" one oscillator with another has received extensive treatment in the literature. Van der Pol and Van der Mark[2] discovered that a relaxation oscillator of natural frequency close to f/n, where n is an integer, could be made to operate at precisely that frequency by injecting a voltage of frequency f. A generator of standard audio frequencies based on this principle and utilizing multi-vibrators was described by Essen[3] in 1936.

The presence of a non-linear conductance or process in the oscillator circuit is an essential feature of the locking phenomena considered in these and a large number of other papers. It is not, therefore, surprising that the inherently non-linear relaxation oscillator has generally proven most suitable for frequency division. The special "quasi-stable" circuits described by Sterky,[4] Miller[5] and Fortescue[6] are, however, exceptions. The usefulness of the multivibrator has been further extended by Davis[7] to obtain non-integral rational ratios.

Recently an urgent need arose for a sine-wave oscillator having a frequency stability of the order of a few parts in 10^6 and capable of operating at a discrete set of frequencies in the 1–3 kc/s range with a spacing not greater than about 3% (continuous tuning was not necessary). Since the only suitable standard readily available was a 1 000 c/s maintained fork, a method of locking in non-integral ratios was demanded. The following device, developed independently by the author, appears to be similar in principle to a method of synchronization described by Tucker.[8]

Most systems of synchronization have in common the injection of the control tone into the oscillator circuit, but Tucker showed that comparison of the control tone and that of the free-running oscillator might be carried out separately and a suitable correction applied to the oscillator frequency. This principle need not, however, be confined to direct synchronization. Indeed it appears to possess outstanding advantages when it is desired to control a sine-wave oscillator at a multiple or sub-multiple of the controlling frequency or in a non-integral rational ratio.

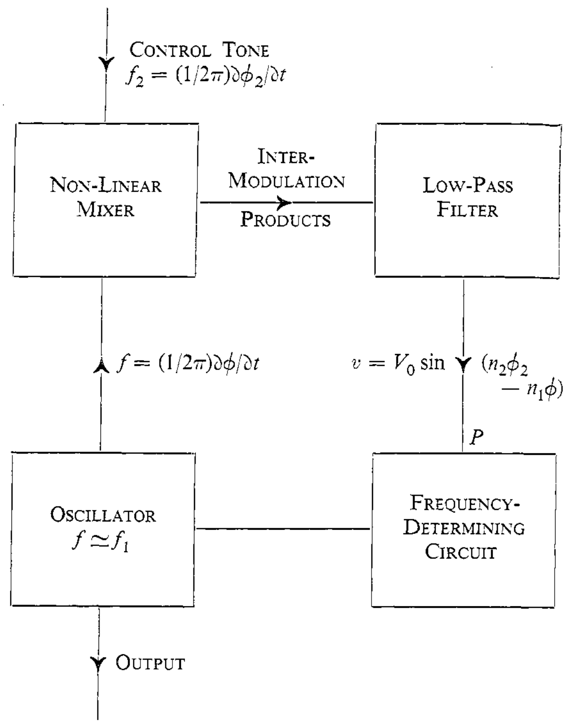


Fig. 1. Principle of frequency-control

THEORY

Referring to Fig. 1, let the oscillator to be controlled and the controlling oscillator have instantaneous phases ϕ and ϕ_2 such that

$$n_2\phi_2 + \text{constant} \equiv n_1\phi_1 \simeq n_1\phi \tag{1}$$

where n_2 and n_1 are integers, ϕ_1 is the desired phase of the oscillator and n_2/n_1 is therefore the control ratio. The corresponding frequencies will be

$$\left.\begin{aligned} f_2 &= (1/2\pi)\partial\phi_2/\partial t \\ f_1 &= (1/2\pi)\partial\phi_1/\partial t \\ f &= (1/2\pi)\partial\phi/\partial t \\ f &\simeq f_1 \end{aligned}\right\} \tag{2}$$

Let both oscillations be applied to a non-linear mixer such that the output voltage contains beats of phase $n_2\phi_2 - n_1\phi$. Beats $m\phi_2 - n\phi$, where m and n are any other pair of integers, will, in general, also be present, but these may be removed by a low pass filter. Then the output voltage of the mixer,

$$v = v_0 \sin (n_2\phi_2 - n_1\phi) \tag{3}$$

is applied to a frequency determining device (such as a reactance valve) coupled to the oscillator.

The oscillator frequency may now be expressed by the equation:

$$f = (1/2\pi)\partial\phi/\partial t = f_1 + \Delta f + kv \tag{4}$$

where $f_1 = (1/2\pi)\partial\phi_1/\partial t$ is the desired controlled oscillator frequency, Δf is the departure of the oscillator from this value due to drift and imperfect initial tuning, and kv is the correction which is applied through the frequency controlling element. Combining (3) and (4) we have:

$$f = f_1 + \Delta f + kV_0 \sin (n_2\phi_2 - n_1\phi) \tag{5}$$

Now f_2 and f_1 are constant by definition. Suppose that there is a solution of equation (5) such that f is independent of time, then the only term involving time-dependent parameters is the last and this, too, must be invariant.

Equations (2) may now be integrated:

$$\left.\begin{aligned} \phi_2 &= 2\pi f_2 t + \alpha \\ \phi &= 2\pi f t + \beta \end{aligned}\right\} \tag{6}$$

Hence

$$n_2\phi_2 - n_1\phi = 2\pi t(n_2 f_2 - n_1 f) + \theta = \text{a constant}$$

where

$$\theta = n_2\alpha - n_1\beta = \text{a constant.}$$

Thus $n_1 f = n_2 f_2 \equiv n_1 f_1$

And, hence, $f = f_1$.

We also find that $\Delta f + kV_0 \sin \theta = 0$, which implies that the oscillator frequency is locked at the desired value of f_1 and that the phase difference $n_2\phi_2 - n_1\phi_1 = \theta$ is adjusted to compensate for Δf. The range of Δf over which locking is maintained is clearly $\pm kV_0$.

This simple treatment does not settle the question of how rapidly changes of Δf will be corrected; indeed it implies that correction will be instantaneous. Closer consideration shows that the controlled oscillator may have the correct mean frequency but may oscillate about this value at some low frequency due to the finite rate at which the oscillator responds to the correction voltage and due to the time delay introduced by the low pass filter circuit. Mathematical treatment of this problem is quite feasible but is hardly profitable since the method and result are intimately related to the properties of the particular system used. However, the general condition which must be fulfilled for stability can readily be stated as follows.

Suppose that the control loop is broken at P and that a voltage $V + v(\omega)$ is applied to the frequency controlling element, where V is a steady component maintaining the correct mean frequency and phase, and $v(\omega)$ is a small alternating component of a low frequency ω. The oscillator will then be phase modulated, leading to an output voltage $V + v'(\omega)$ from the mixer and low pass filter circuits. If the equation

$$v'(\omega) = v(\omega) \tag{7}$$

is satisfied both in magnitude and phase for any particular value of ω, phase oscillation will result when the control loop is closed and the steady state solution of equation (5) will not be realized. The problem is, in fact, one analogous to that of feedback amplifier design in which Nyquist's theorem may appropriately be applied.

A PRACTICAL ARRANGEMENT

In Fig. 2, the circuit is given of a practical arrangement which has met the need expressed earlier, for a generator of stable audio frequencies. An *R-C* oscillator of the Wien Bridge type with lamp stabilization is used and its frequency is controlled through the reactance valve V_3 which has only a relatively small effect on the frequency (a few %). The output from the oscillator is amplified by V_1 to a level about 50 V r.m.s. This is applied through a limiting resistor to the control grid of V_2 a 6L7 pentagrid mixer valve. Similarly, the voltage from the 1 000 c/s tuning-fork is stepped up to a level of about 50 V, limited and applied to the third grid of V_2. Thus the input waveforms of the first and third grids of V_2 are both rectangular. (That portion of the negative grid swing beyond anode current cut-off is clearly ineffective.)

The anode current of V_2 contains very many intermodulation products and all but the very low frequency components are attenuated by a simple low pass filter composed of the anode load resistor in parallel with a capacitance giving a time constant of 0·04 sec or a characteristic frequency of 4 c/s. This load is directly coupled to the grid of the reactance valve V_3.

The performance of this arrangement was observed by connecting the X and Y plates of a cathode ray oscillograph at the points X and Y in Fig. 2. It was found that the *R-C* oscillator was locked whenever it was tuned close to a frequency f_1 where $n_1 f_1 = n_2 f_2$; n_1 and n_2 might have integral values as high as 30. Small externally induced fluctuations of f led to very rapid readjustment (of the order of 0·1 sec). There was no tendency to oscillation of frequency about the correct value, in the manner considered earlier, but this could be produced by introducing an additional *R-C* filter between the mixer and the reactance valve. This is to be expected since the oscillator will exhibit some inertia in responding to changes of control voltage and this, with a two-stage filter network, can lead to the conditions needed to satisfy equation (7).

Although locking occurred at a very large number of frequencies over a wide range (from about 100 c/s–10 kc/s), the component values were chosen to give optimum performance in a much more restricted region. The range 2·0–2·5 kc/s was examined in detail and it was found that locking occurred at the frequencies shown in the Table. Those frequencies marked with an asterisk were securely locked and could be maintained indefinitely. The remainder were locked but not sufficiently well to remain so without retuning the oscillator at intervals. The locking range $=\Delta f$ was greater than 0·2% of f_1 at half of the controlled frequencies and was 1·5% in the best case (at 2·0 kc/s). The harmonic distortion of the oscillator output and the amplitudes of the 1 000 c/s note and other unwanted frequencies were negligible.

However, in a few cases, particularly at the higher values of n_2 and n_1, there was evidence of slight phase modulation of the controlled oscillator. This is not to be confused with the type of instability mentioned earlier. Consideration of the functioning of the mixer shows that, when locking has occurred, its anode current contains only one set of frequencies: the highest common factor F of the two frequencies f_1 and f_2, viz.,

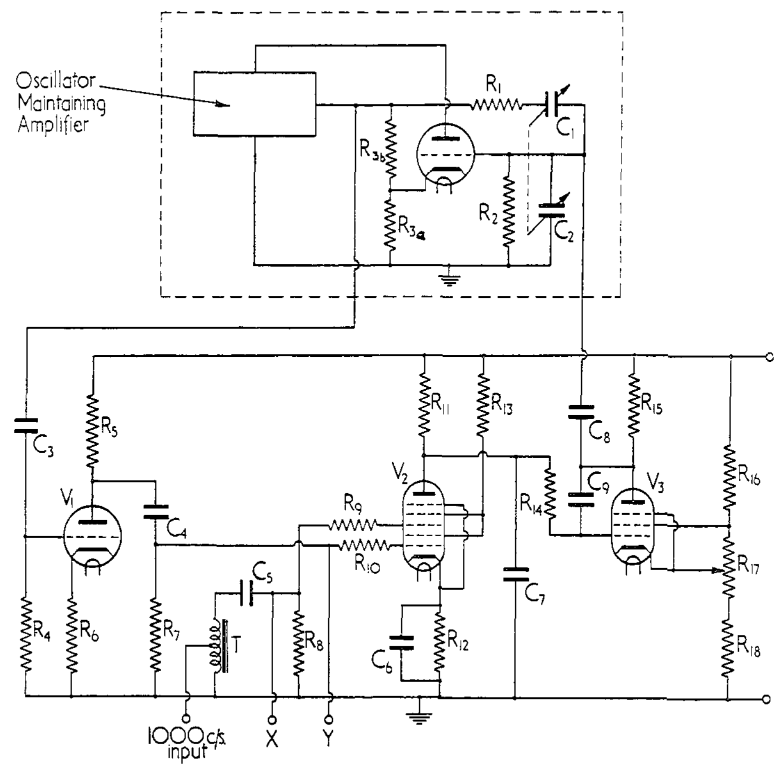
$$F = f_2/n_1 = f_1/n_2 \qquad (8)$$

Frequencies at which locking occurred within 2·0–2·5 kc/s range

1 c/s	n_2	n_1	1 c/s	n_2	n_1
*2 000·0	2	1	2 300·0	23	10
2 100·0	21	10	2 307·7	30	13
2 111·1	19	9	*2 333·3	7	3
*2 125·0	17	8	2 357·1	33	14
*2 142·9	15	7	2 363·6	26	11
2 166·7	13	6	2 375·0	19	8
2 181·8	24	11	*2 400·0	12	5
*2 200·0	11	5	*2 428·6	17	7
2 222·2	20	9	2 444·4	22	9
*2 250·0	9	4	2 454·5	27	11
2 272·7	25	11	*2 500·0	5	2
*2 285·7	16	7			

* Frequencies locked and maintained.

together with the harmonics 2F, 3F . . . and a steady component. It is the latter which maintains the oscillator locked, the others being filtered out. However, when n_1 and n_2 are large, F is low and may not be completely eliminated in which case it might be passed on to the reactance valve. Other effects are also possible should a third frequency—for example that of the mains—be present in the mixer circuit.



Component values

$R_1 = R_2$ = 76·8 kΩ (for 1·4–5·1 kc/s range)
R_{3a} = Oscillator stabilizing lamp (3rd arm of Wien Bridge)
R_{3b} = 4th arm of bridge
R_4 = 500 kΩ
R_5 = 220 kΩ
R_6 = 5 kΩ
R_7 = 2 MΩ
R_8 = 2 MΩ
R_9 = 560 kΩ
R_{10} = 560 kΩ
R_{11} = 20 kΩ
R_{12} = 3·3 kΩ
R_{13} = 15 kΩ
R_{14} = 56 kΩ
R_{15} = 2 MΩ
R_{16} = 10 kΩ
R_{17} = 100 Ω wire wound variable
R_{18} = 22 kΩ

$C_1 = C_2$ = 400–1 300 μμF
C_3 = 750 μμF
C_4 = 0·1 μF
C_5 = 0·1 μF
C_6 = 50 μF
C_7 = 2 μF
C_8 = 150 μμF
C_9 = 30 μμF

V_1 = 6J5
V_2 = 6L7
V_3 = EF39

T = 2/1 transformer

Fig. 2. A practical circuit for frequency controlling an R.C. oscillator

CONCLUSION

A method of controlling an oscillator at a multiple or a sub-multiple of the controlling frequency, or in a non-integral rational ratio, has been described with particular reference to the provision of a discrete set of highly stable audio frequencies. The practical arrangement is capable of further development by separation of the different functions of the mixer valve. For example, the sine-wave controlling tone and the output of the oscillator might be used to produce two synchronized trains of pulses operating a "gate" circuit such that only coincident or partially coincident pulses would contribute to the output current. The d.c. component of the latter could then be used to effect control of the oscillator circuit.

Although the work described has been concerned exclusively with audio oscillators, the principle is clearly a more general one, and it is possible that u.h.f. or even microwave oscillators could be frequency-controlled in a similar manner.

REFERENCES

(1) APPLETON, E. V. *Proc. Camb. Phil. Soc.*, **21,** p. 231 (1922)
(2) VAN DER POL and VAN DER MARK. *Nature, Lond.*, **120** p. 363 (1927).
(3) ESSEN, L. *J. Sci. Instrum.*, **13,** p. 9 (1936).
(4) STERKY, H. *Proc. Inst. Radio Engrs, N.Y.*, **25,** p. 115[illegible] (1937).
(5) MILLER, R. L. *Proc. Inst. Radio Engrs, N.Y.*, **27,** p. 44[illegible] (1939).
(6) FORTESCUE, R. L. *J. Instn Elec. Engrs*, **84,** p. 693 (1939)
(7) DAVIS, K. H. *Bell Lab. Record*, **24,** p. 114 (1948).
(8) TUCKER, D. G. *Electronic Engng*, **16,** p. 26 (1943).

SPECIAL REPORT

Report of Discussion on Calibration Line Drift in Spectrographic Analysis—Sheffield, February 1950

The Industrial Spectroscopy Group of The Institute of Physics and the Sheffield Spectrographic Discussion Group held a joint meeting in the University of Sheffield on 17 February, 1950, to discuss calibration line drift in spectrochemical analysis. The papers and the discussion on them are summarized in this report.

The meeting was opened by Mr. H. R. Clayton (British Aluminium Co. Ltd.), who summarized a number of contributions. Short papers were presented by: Mr. H. R. CLAYTON, Mr. R. J. WEBB (Ministry of Supply, Woolwich), Mr. R. DIXON (William Jessop and Sons Ltd., Sheffield), Dr. R. O. SCOTT (Macaulay Institute, Aberdeen) and Mr. W. E. VAN R. DE JONG (N.V. Hollandsche, Metallurgische Bedrijven).

Mr. Dixon presented results obtained by the Sheffield Spectrographic Discussion Group: and the contribution made by Mr. Clayton summarized work in his own laboratories as well as the laboratories of Dr. Addink (N.V. Philips Gloeilampenfabrieken, Eindhoven), Mr. Dunwoody (Northern Aluminium Co. Ltd.), Mr. S. Kipling (Wolsey Motors Ltd.), Mr. E. van Someren (Murex Welding Processes Ltd.), and the Post Office Engineering Department.

The short papers dealt with different aspects of calibration drift and covered a variety of types of material and excitation. The discussion which followed was very active and as it dealt with many points not covered in the introductory papers the following brief account summarizes both the papers and the discussion irrespective of the source of the information.

DRIFT ARISING IN THE DISCHARGE

Evidence was produced showing that with d.c. arc excitation of powdered mineral samples in craters in graphite electrodes, variations in the depth or diameter of the crater influence the index point of the calibration graph and that the tightness with which the powder is packed into the crater is also of great importance. The effect of background variations in this type of analysis was also stressed, and graphs were presented to show the advantages of background compensation, particularly when wide ranges of concentrations (300-fold) are to be covered. Under these conditions curves uncorrected for background show both lateral and angular shift whilst those to which a background correction is