An autocorrelogram computer

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A comparatively simple and inexpensive instrument capable of computing autocorrelation functions is described. It was built for the analysis of irregularity in yarn, roving and slivers in the textile industry, but it should find application in a much wider field in cases where the high accuracy of the expensive precision computer is not required. The instrument also lends itself to the computation of cross-correlation functions.

Principles of operation and construction are given together with data on performance. The yarn irregularity is recorded on a magnetic tape as a frequency-modulated signal and use is made of a modified d.c. watthour meter for carrying out the multiplying and integrating operations.

1. INTRODUCTION

During recent years, there has been increasing use of the autocorrelation function for detecting weak periodic signals hidden by random components of large amplitude. This type of problem has arisen in a number of widely separated fields such as communication engineering,(1) meteorology,(2) marine physics,(3, 4) medicine(1) and numerous other subjects.(5, 6) It arises in the textile industry in an acute form in relation to the analysis of the irregularity in cross-section of yarns.

Periodic components in the irregularity of yarns can give rise to highly objectionable patterning in the fabric woven at cloth widths which bear some relationship to the wavelength of the periodic component. Electronic yarn testers of different types(7) are being used in a number of testing laboratories associated with mills, but their usefulness is limited to providing a figure for the magnitude of the random variations in the yarn cross-section, and it is only when the yarn is of very poor quality that a periodic variation can be picked out with any degree of certainty. Periodicity in the yarn can be looked for in a qualitative manner by means of "blackboard tests," in which the yarn is wound on to a board and the presence of patterning can be seen if it is sufficiently marked. This type of test is, however, highly subjective in its nature and very time-consuming if a comprehensive assessment is required.

In order to characterize the output from a set of spindles, a large number of yarn samples needs to be taken. A comparatively cheap computer is, therefore, required which will analyse the output signal from a yarn irregularity instrument in as short a time as possible. This problem has been tackled before(8) using a photographic recording and optical method, but the method described here relies almost entirely upon electronic means and differs from the optical method in many important respects.

2. THE AUTOCORRELATION FUNCTION

The autocorrelation function of a time series $f(t)$ is defined as

$$\Phi(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T f(t).f(t-\tau)dt \qquad (1)$$

This function is the one normally used in communication engineering. $\Phi(\tau)$ assumes the form of an autocorrelation function as used in statistics when $f(t)$ has zero mean and unit variance, and it can give us additional information about the time series $f(t)$, as can be shown in the following. By way of an illustrative example let us assume that $f(t)$ consists of two terms: a periodic and a random one.

$$f(t) = F(t) + f_0 \cos \omega t \qquad (2)$$

where $F(t)$ is a random time series and f_0 is much smaller than the r.m.s. value of the random component.

It can be shown(9) that

$$\Phi(\tau) = \Phi_{ss}(\tau) + \Phi_{sr}(\tau) + \Phi_{rs}(\tau) + \Phi_{rr}(\tau) \qquad (3)$$

where Φ_{ss} is the autocorrelation function of the periodic component, Φ_{sr} and Φ_{rs} are the cross-correlation functions of the periodic and random components and Φ_{rr} is the autocorrelation function of the random component. Owing to the incoherence of the periodic and random components, it can be shown that Φ_{rs} and Φ_{sr} will vanish and also that Φ_{rr} will approach zero as τ increases. Also

$$\Phi_{ss}(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_0^T (f_0 \cos \omega t).[f_0 \cos \omega(t-\tau)]dt = \frac{f_0^2}{2}\cos \omega\tau \qquad (4)$$

It follows that

$$\Phi(\tau) = \Phi_{rr}(\tau) + (f_0^2/2) \cos \omega\tau \qquad (5)$$

yielding a function as shown in Fig. 1.

It is possible to improve the periodic-to-random component

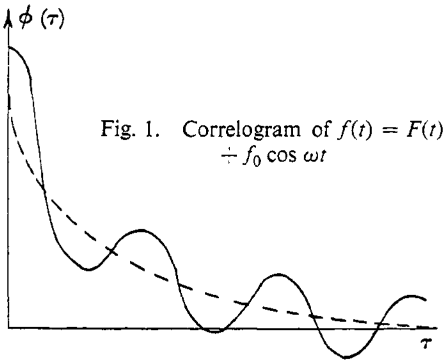

Fig. 1. Correlogram of $f(t) = F(t) + f_0 \cos \omega t$

ratio of the original function $f(t)$ quite considerably by operating on it in this way and $\Phi(\tau)$ may show a periodic component which cannot be detected in $f(t)$.

3. BASIC LAYOUT OF THE COMPUTER

As can be seen from Section 2, the tasks of the computer, given a time series in the form of a continuous signal $f(t)$, are as follows:

(i) arrange that the function $f(t-\tau)$ is available for operation at the same time as $f(t)$;
(ii) form the product $f(t).f(t-\tau)$;
(iii) integrate the product over a time interval T;
(iv) record the result thus obtained.

The method of obtaining the signal in electrical form depends, naturally, on the type of phenomenon observed. In the case of a yarn, various methods are available.(7) Assuming that

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such a signal is available, the principle of the computer is as shown in the block diagram of Fig. 2.

The signal is recorded on a magnetic tape using frequency modulation and afterwards played back through two heads. The length of the tape between the two heads can be varied by arranging for a loop of tape to be pulled out as shown in Fig. 3 and the delay between $f(t)$ and $f(t-\tau)$ thus obtained.

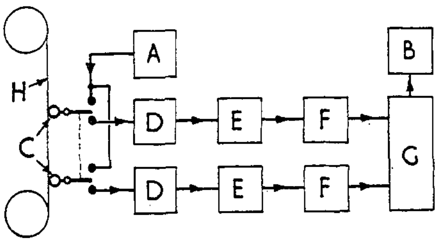

Fig. 2. Block diagram of the computer

A, modulator; *B*, counter; *C*, heads; *D*, preamplifier; *E*, demodulator; *F*, driver amplifier; *G*, multiplier and integrator; *H*, tape.

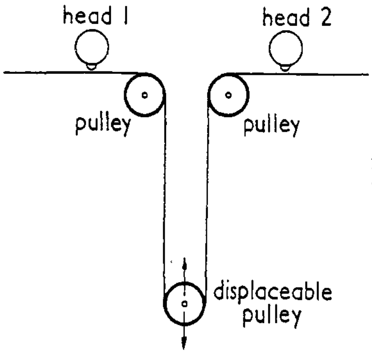

Fig. 3. Arrangement for obtaining variable separation

The outputs from the two heads are fed into identical channels, consisting of preamplifier, demodulator and output stages. The outputs are used to drive the multiplier-integrator system. This is a modified d.c. watthour meter as shown in Fig. 4. It is basically a d.c. motor with an ironless armature, the driving torque of which is proportional to the field flux (i.e. to the field current I_f) and the armature current I_a, thus:

$$T_d \propto I_a . I_f \tag{6}$$

An aluminium disk *D* (Fig. 4) mounted on the same axis as the armature revolves between the poles of a permanent magnet *M* and provides a retarding torque which is proportional to the speed. When frictional forces (due to bearings and air damping) are small, equilibrium is established when the driving torque equals the retarding torque. The speed of revolution will then be

$$n = k . I_a . I_f \tag{6a}$$

where k is a constant. The relationship holds well over a range of speed of 100 to 1 as shown in Fig. 7. It can be seen that at very low speeds, where the effect of friction becomes comparable with that of the velocity braking, the relationship ceases to be a linear one.

If the number of revolutions is counted over a time interval T

$$N = \frac{1}{T} \cdot \int_0^T n(t)dt = \frac{k}{T} \int_0^T I_a . I_f dt \tag{7}$$

If I_a and I_f are made proportional to the signal $f(t)$ and its displaced value respectively,

$$N = \frac{K}{T} \cdot \int_0^T f(t) . f(t-\tau)dt \tag{8}$$

(In actual fact, additional factors enter as will be shown in Section 6). Hence it can be seen that the required autocorrelation function is

$$\Phi(\tau) \propto \lim_{T\to\infty} N(\tau) \tag{9}$$

i.e. the number of revolutions of the watthour meter counted over a time interval T is approximately proportional to the autocorrelation function of the time series analysed. The accuracy improves with increased T and when choosing T a compromise has to be made to keep the time for testing down to a minimum.

The counting is done by means of holes drilled into the braking disk. The holes pass between a light source *L* and a photocell *P* (Fig. 4) and the pulse from the photocell is amplified to actuate an electromagnetic counter.

4. THE ELECTRONIC CIRCUIT

The detailed circuit diagram is shown in Fig. 5. The tape recorder modulator and demodulator circuits are based on existing designs[10] and are given here for the sake of completeness. An input voltage variation of ± 5 V to the modulator results in an output of sensibly constant amplitude and a frequency variation of 1 to 5 kc/s, varying linearly with the input voltage. The demodulators are preceded by audio-frequency amplifiers and followed each by a two-stage d.c. amplifier. The h.t. supplies to the output stages are stabilized each by a two-stage shunt regulating system ($V_{18} V_{19} V_{30}$ and

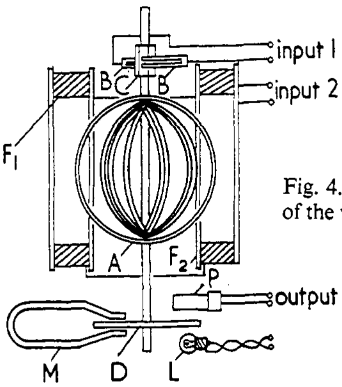

Fig. 4. Basic layout of the watthour meter

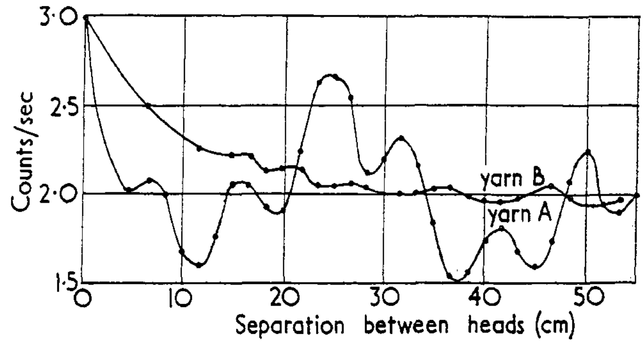

Fig. 6. Two examples of correlograms

$V_{22}V_{23}V_{31}$). This method requires a lower h.t. supply than series-shunt stabilization and was found to be adequate. The other stabilized power supply (V_{24} to V_{26}) provides h.t. for the modulator-demodulator system and is of conventional

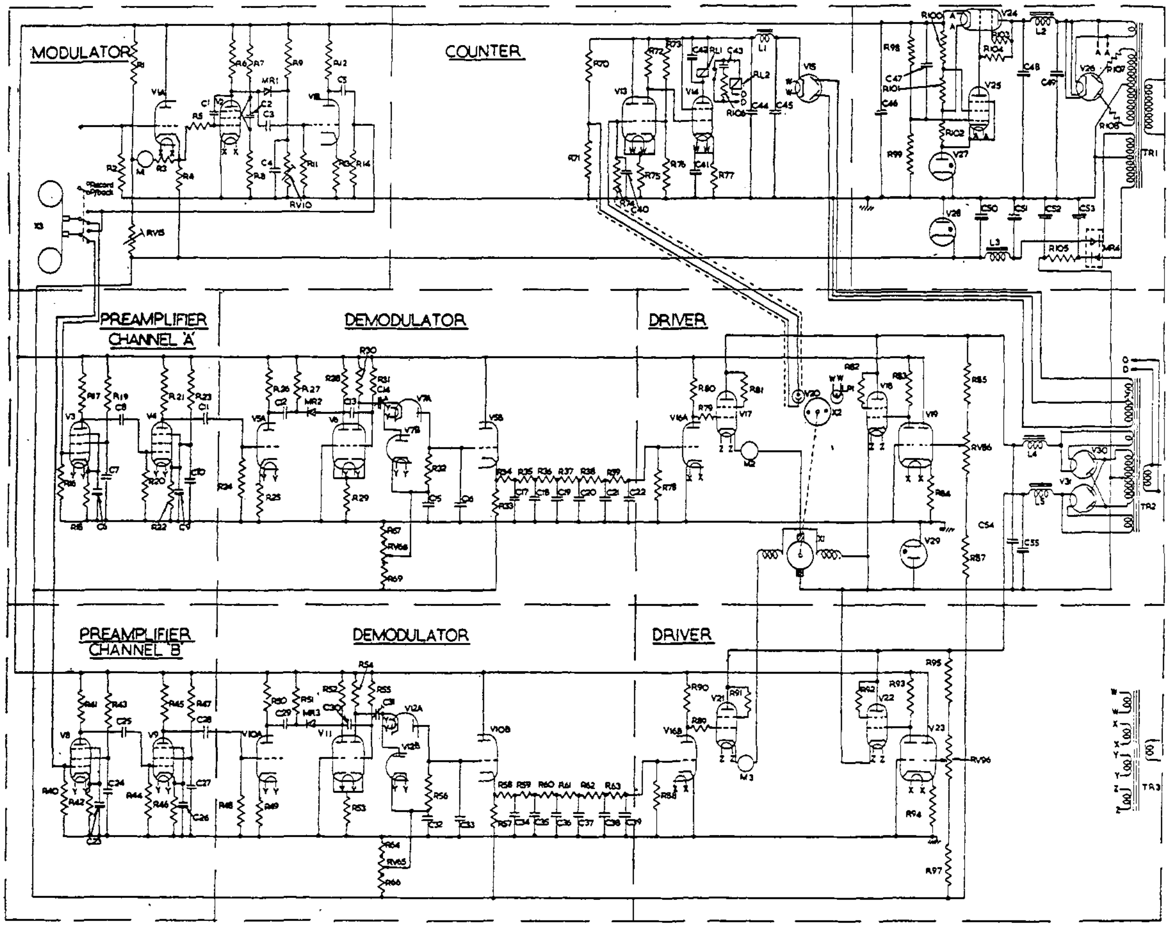

Fig. 5. Circuit diagram of the computer

(All resistors ½ W unless otherwise stated)
R_1 = 100 kΩ
R_2 = 1 MΩ
R_3, R_4 = 68 kΩ
R_5, R_6, R_7 = 100 kΩ
R_8 = 3·3 kΩ
R_9 = 33 kΩ
RV_{10} = 10 kΩ
R_{11} = 470 kΩ
R_{12} = 68 kΩ
R_{13} = 3·3 kΩ
R_{14} = 1 MΩ
RV_{15} = 10 kΩ
R_{16} = 150 kΩ
R_{17} = 470 kΩ
R_{18} = 2·2 kΩ
R_{19} = 470 kΩ
R_{20} = 1 MΩ
R_{21} = 470 kΩ
R_{22} = 2·2 kΩ
R_{23}, R_{24} = 1 MΩ
R_{25} = 3·3 kΩ
R_{26} = 68 kΩ
R_{27} = 220 kΩ
R_{28} = 47 kΩ
R_{29} = 4·7 kΩ
R_{30} = 47 kΩ
R_{31} = 3·3 MΩ
R_{32} = 100 kΩ
R_{33} = 68 kΩ
R_{34} = 3·3 kΩ
R_{35} = 6·8 kΩ
R_{36} = 8·2 kΩ
R_{37}, R_{38} = 15 kΩ
R_{39} = 22 kΩ
R_{40} = 150 kΩ
R_{41} = 470 kΩ
R_{42} = 2·2 kΩ
R_{43} = 470 kΩ
R_{44} = 1 MΩ
R_{45} = 470 kΩ
R_{46} = 2·2 kΩ
R_{47}, R_{48} = 1 MΩ
R_{49} = 3·3 kΩ
R_{50} = 68 kΩ
R_{51} = 220 kΩ
R_{52} = 47 kΩ
R_{53} = 4·7 kΩ
R_{54} = 47 kΩ
R_{55} = 3·3 MΩ
R_{56} = 100 kΩ
R_{57} = 68 kΩ
R_{58} = 3·3 kΩ
R_{59} = 6·8 kΩ
R_{60} = 8·2 kΩ
R_{61}, R_{62} = 15 kΩ
R_{63} = 22 kΩ
R_{64} = 4·7 kΩ
RV_{65} = 25 kΩ
R_{66} = 100 kΩ
R_{67} = 4·7 kΩ
RV_{68} = 25 kΩ
R_{69} = 100 kΩ
R_{70}, R_{71} = 22 kΩ
R_{72} = 100 kΩ
R_{73} = 220 kΩ
R_{74} = 10 MΩ
R_{75} = 2·2 kΩ
R_{76} = 10 kΩ
R_{77} = 33 kΩ
R_{78} = 1 MΩ
R_{79} = 1 kΩ
R_{80} = 470 kΩ
R_{81}, R_{82} = 100 kΩ
R_{83} = 470 kΩ
R_{84} = 2·2 kΩ
R_{85} = 1 MΩ
RV_{86} = 500 kΩ
R_{87} = 220 kΩ
R_{88} = 1 MΩ
R_{89} = 1 kΩ
R_{90} = 470 kΩ
R_{91}, R_{92} = 100 Ω
R_{93} = 470 kΩ
R_{94} = 2·2 kΩ
R_{95} = 1 MΩ
R_{96} = 500 kΩ
R_{97} = 220 kΩ
R_{98} = 330 kΩ
R_{99} = 470 kΩ
R_{100}, R_{101} = 4·7 kΩ
R_{102} = 1 W, 15 kΩ
R_{103} = 100 Ω
R_{104} = 470 kΩ
R_{105} = 3 W, 6·2 kΩ
R_{106} = 22 kΩ
R_{107}, R_{108} = 100 Ω

(All voltages 350 V working unless otherwise stated)
C_1 = 0·001 μF
C_2, C_3 = 0·01 μF
C_4, C_5 = 0·05 μF
C_6 = 25 μF, 25 V
C_7 = 1·0 μF
C_8 = 500 pF
C_9 = 25 μF, 25 V
C_{10} = 1·0 μF
C_{11} = 500 pF
C_{12} = 0·01 μF
C_{13} = 0·001 μF
C_{14} = 175 pF, var.
C_{15} = 0·01 μF
C_{16} = 0·005 μF
C_{17}–C_{22} = 0·01 μF
C_{23} = 25 μF, 25 V
C_{24} = 1·0 μF
C_{25} = 500 pF
C_{26} = 25 μF, 25 V
C_{27} = 1·0 μF
C_{28} = 500 pF
C_{29} = 0·01 μF
C_{30} = 0·001 μF
C_{31} = 175 pF var.
C_{32} = 0·01 μF
C_{33} = 0·005 μF
C_{34}–C_{39} = 0·01 μF
C_{40} = 330 pF
C_{41} = 32 μF
C_{42} = 8 μF
C_{43} = 0·5 μF, 450 V
C_{44}, C_{45} = 16 μF, 450 V
C_{46} = 32 μF, 450 V
C_{47} = 0·5 μF, 450 V
C_{48}, C_{49} = 8 μF, 600 V
C_{50}, C_{51} = 16 μF, 450 V
C_{52}, C_{53} = 8 μF, 450 V
C_{54}, C_{55} = 8 μF, 600 V

V_1 = CV455 (12AT7)
V_2 = EF80
V_3, V_4 = CV138 (6AM6)
V_5 = CV455 (12AT7)
V_6 = CV492 (12AX7)
V_7 = CV140 (6AL5)
V_8, V_9 = CV138 (6AM6)
V_{10} = CV455 (12AT7)
V_{11} = CV492 (12AX7)
V_{12} = CV140 (6AL5)
V_{13} = CV491 (12AU7)
V_{14} = CV2136 (6BW6)
V_{15} = CV493 (6X4)
V_{16} = CV455 (12AT7)
V_{17}, V_{18} = CV391 (5B/255M)
V_{19} = CV858 (6J6)
V_{20} = CV2133 (90CG)
V_{21}, V_{22} = CV391 (5B/255M)
V_{23} = CV858 (6J6)
V_{24} = CV2721 (EL81)
V_{25} = CV138 (6AM6)
V_{26} = CV493 (6X4)
V_{27} = CV287 (QS150/15)
V_{28} = 90C1
V_{29} = CV287 (QS150/15)
V_{30}, V_{31} = 5Z4

MR_1, MR_2, MR_3 = 1 mA Inst. rect.
MR_4 = DRM2B (Brimar)

M_1 = meter, 100–0–100 μA
M_2, M_3 = meter, 100 mA full-scale deflexion

RL_1 = high speed relay 1 kΩ, 10 mA
RL_2 = Veeder-Root magnetic counter

LP_1 = signal lamp, 6·5 V, 0·3 A

X_1, X_2 = watthour meter with light chopping disk
X_3 = tape deck, Wearite type 2C

L_1, L_2, L_3 = 15 H, 50 mA
L_4, L_5 = 9 H, 100 mA

T_1 = 350–0–350 V, 70 mA
150–0–150 V, 70 mA
6·3 V, 2 A
T_2 = 350–0–350 V, 200 mA
250–0–250 V, 50 mA
2 × 5 V, 2 A
T_3 = 6·3 V, 2 A; 6·3 V, 2·5 A
6·3 V, 3 A; 6·3 V, 3·5 A

design. The counting circuit (V_{13} and V_{14}) has a separate power supply since it does not require any stabilization and would superimpose undue load on the regulated supply.

5. METHOD OF OPERATION

The instrument operates in the following manner. The signal to be analysed is recorded *via* the modulator on to the tape. In our case the electrical signal was obtained by using a Fielden–Walker type yarn-irregularity tester,[11] but various other existing instruments[7] could serve the purpose as well. The recording is then played back at a certain fixed separation of the tape corresponding to a time delay τ and the number of counts $N(\tau)$ thus obtained noted. By altering the tape length between the heads (i.e. the effective separation of the heads), different counts are successively obtained; these values are subsequently plotted and give the function $N(\tau)$. This function differs from the required correlation function $\Phi(\tau)$ by being raised along the ordinate and by having a different scale. Generally the function $N(\tau)$ gives all the information required, i.e. it indicates any existing periodic component and its relative magnitude as compared with the value $N(O)$. Thus, for instance, Fig. 6 shows the results obtained for two different yarn samples, yarn *A* having two pronounced periodic components (with a wavelength of 7·5 and 25 cm, respectively), whereas yarn *B* has no periodic component. This is, in most cases, the only information required. For some cases, where the exact function $\Phi(\tau)$ is required, an additional measurement has to be performed which enables us to normalize the function $N(\tau)$.

6. NORMALIZATION OF RESULTS

Assume that $I(t)$—that is the current through either the stator or the rotor of the watthour meter—is a linear function of the signal $f(t)$ to be analysed, i.e.

$$I(t) = I_0 + m.f(t) \quad \text{and} \quad I(t-\tau) = I_0 + m.f(t-\tau) \tag{10}$$

where m is a constant. Substituting from equation (10) into equation (7) we have

$$N(\tau) = kI_0^2 + kI_0\frac{m}{T}\cdot\int_0^T [f(t) + f(t-\tau)]dt + k\frac{m^2}{T}\cdot\int_0^T f(t)f(t-\tau)dt \tag{11}$$

The expression for $\Phi(\tau)$ is contained in the third term. The values k, m and I_0 are constants and can be evaluated from the characteristic curves of the instrument, which are shown in Fig. 7. The latter two constants are made identical for both field and armature by adjusting C_{14}, RV_{68} and C_{31}, RV_{65} in the circuit (see Fig. 5), but the following reasoning will, of course, hold when the constants are different. In order to evaluate the second term in equation (11), a further series of measurements is necessary. Suppose we feed one of the coils (field or armature) with the constant current I_0 and feed the other coil with the signal from one of the heads. We will then have

$$N_0(\tau) = \frac{k}{T}\int_0^T I_0 . [I_0 + mf(t-\tau)]dt \tag{12}$$

Multiplying and substituting equation (12) into equation (11) we obtain for the corrected zero level

$$N^*(\tau) = N(\tau) - [N_0(\tau) + N_0(O) - kI_0^2] \tag{13}$$

It was found that, in most of the practical tests carried out, the assumption could be made that

$$N_0(\tau) \simeq N_0(O) \quad \text{i.e.} \quad \int_0^T f(t)dt \simeq \int_0^T f(t-\tau)dt$$

This means that sufficiently accurate results can be obtained by making one measurement only and determining $N_0(O)$ as outlined above, and by using, instead of equation (13), the simplified form

$$N^*(\tau) = N(\tau) - [2N_0(O) - kI_0^2] \tag{13a}$$

Since

$$\Phi(\tau)/\Phi(O) = N^*(\tau)/N(O) \tag{14}$$

by choosing the ordinate $N^*(O)$ as unity the normalized correlogram can thus be obtained.

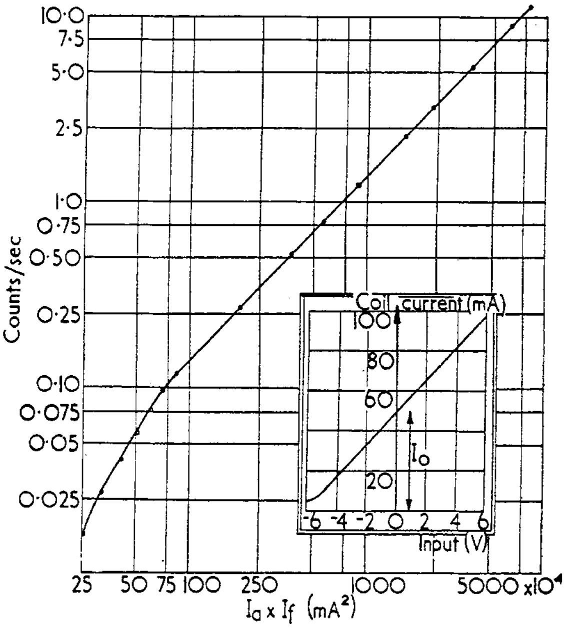

Fig. 7. Counts per second as a function of current through coils. *Inset:* coil current as a function of input to modulator

$n = k . I_a . I_f$
$k = 1·3 \times 10^{-3}$ counts per second per mA^2

It is clear then, from the foregoing, that the instrument is capable of computing the following expressions:

$$N^*(\tau)/km^2 = \frac{1}{T}\int_0^T f(t).f(t-\tau)dt$$

$$N^*(O)/km^2 = \frac{1}{T}\int_0^T f^2(t)dt$$

and the ratio of these two expressions is the value of the autocorrelation function as used in statistical theory, when, as is usually the case, the mean value of $f(t)$ is zero. If the mean of $f(t)$ is not zero, it can be calculated from equation (12) and the appropriate correction in the autocorrelation function made.

7. CONSTRUCTIONAL DETAILS

(i) *The tape deck.* The deck used is a Wright and Weaire type C modified to incorporate the pulley arrangement shown in Fig. 3. This deck has two reproducing heads and is designed for simultaneous double-track recording using standard ¼ in. tape.

The deck can also be equipped with a continuous loop casette, manufactured by the same firm, which can be used with advantage for short separations (τ) only. When greater lengths are to be analysed, a normal two-reel operation is more reliable, requiring rewinding after each measurement. The maximum time separation also determines the required integration time T, which should be much greater than the former. Good results were obtained with a ratio of 10 : 1.

The displaceable pulley can be moved along a pole mounted at right angles to the line joining the two heads and in the plane of the tape deck. This pole has holes 2·5 cm apart to hold the spindle of the pulley and its length is 75 cm, with an attachment pole also 75 cm long. The maximum separation was thus 3 m. By recording at a slower tape-speed the scale of the separations could be altered and, hence, higher maximum separation achieved. The tape deck used has two speeds, namely 3·75 and 7·5 in./sec.

(ii) *The watthour meter.* The basic layout of this unit is shown in Fig. 4 and the following details should be noted:

The two field-coils F_1 and F_2 are series-connected and wound on a former of 2½ in. internal diameter, 2 in. thick, each coil having 6500 turns of 30 s.w.g. enamelled copper wire.

The armature A consists of three flat coils, each wound in planes displaced at 120° and Δ-connected, the junctions being brought out to the commutator C consisting of three silver segments. The brushes B are of spring steel with silver-plated tips. Each of the three rotor coils has 1500 turns of 44 s.w.g. enamelled copper wire.

The braking disk is made of 16 s.w.g. aluminium and has a diameter of 3 in. This disk also carries two holes which enable pulses of light to fall on the photocell. This number of holes results in a maximum of 10 counts per second at full speed of the meter. Whilst a higher number of holes would have increased the accuracy of the instrument, the limit was set by the frequency response of the counting circuit, in particular the magnetic counter. The counter used was a Veeder–Root type.

8. ACCURACY OF THE COMPUTER

Being a d.c. computer, the instrument requires a warming-up time. It was found that about ten minutes sufficed, provided the watthour meter was kept running by feeding the modulator output directly into the two playback channels so that the currents through the meter coils were about 50% of the maximum value. With this precaution the instrument performed as follows:

Repeatability of counts: not more than 0·6% change from day to day on one and the same record.

Repeatability of correlograms: not more than 5% change from day to day. This latter figure is, of course, much higher than the former, since according to equation (13) the correlogram is obtained as the difference of two quantities, which are very nearly equal.

9. COMPUTATION OF CROSS-CORRELATION FUNCTIONS

The instrument can also be used for the computation of cross-correlation functions. Since the deck used is capable of simultaneous double-track recording, the only addition required is a second modulator unit to record the second signal.

10. CONCLUSION

The instrument, built according to the principles outlined above, was found to perform satisfactorily. Research work is at present being carried out to apply the correlogram to the quality control of yarns and the results will be published at a later date.

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Vibrating plate method of producing powder ridges in a sound field

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A circular brass plate vibrating in the mode which includes one nodal circle, will, at suitable intensities, produce circular powder ridges on a horizontal surface situated a few millimetres below it. This rapid method of experiment makes it possible to study the numerous variables which affect the formation of powder ridges in an oscillating fluid. Size of particle is, for instance, of fundamental importance. Intensity effects are relatively small.

ORIGIN OF THE EXPERIMENTS

When a Chladni plate is set into vibration by means of solid carbon dioxide[(1)] it is usual to support it on small rubber studs, thus leaving a narrow air-gap between the plate and the bench below. In the course of experiment sand accumulates beneath the plate, and on one occasion it was noticed that on some of this sand ridges were clearly visible.[(2)] The writer had been experimenting for some years before this

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