An analogue computing circuit for the evaluation of the ratio of two slowly-varying potentials

By R. L. Gordon, B.Sc., Ph.D., Graduate I.E.E., A.Inst.P., Safety in Mines Research Establishment, Sheffield

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An electronic circuit is described which accurately computes the ratio of two slowly-varying potentials, displaying the result as a meter deflexion. Feedback is used to reduce the sensitivity of the circuit to valve characteristic variations, and an accuracy of better than 1% of full-scale deflexion of the meter has been obtained within wide input limits. Computations of reciprocals and products may be performed by similar means. Several possible applications of the device are mentioned.

In electrical analogue computing the problem of multiplication and division has always been somewhat troublesome. The solution of the problem has generally utilized either small servo-mechanisms and mechanically-driven linear potentiometers or the *non-linear* characteristics of thermionic valves or of metal rectifiers. The present device is entirely electronic in character. It is so designed that valve characteristics have only a second-order effect upon its stability and accuracy.

BASIC PRINCIPLES

The central element of the device is the "Miller integrator," originated by the late A. D. Blumlein.[(1–3)] In this circuit, shown in skeleton form in Fig. 1, the potential across the anode load R_{16} is the time-integral of the potential E fed to the grid resistor R_{19}. For a constant input signal, therefore, the anode potential executes a linear "run-down" over the working range of the valve; the rate of fall of anode potential is directly proportional to E. The circuit is switched cyclically, the "run-down" being repeated once per cycle.

The present circuit uses the Miller integrator in two forms:

(*a*) The "Sanatron" circuit developed by Williams and Moody.[(2,4)] This is arranged to retrigger itself at the end of the run-down, the amplitude of which is maintained constant. The rate of run-down is made directly proportional to the denominator variable by feeding this to the grid resistor, so that the *time* of each run-down is inversely proportional to the denominator.

(*b*) A conventional Miller circuit, in which the rate of run-down is made proportional to the numerator variable. This circuit is switched by a signal from the Sanatron, so that the period of run-down is made equal to that of the Sanatron circuit. Thus the Miller valve anode potential falls, during each cycle, by an amount proportional to the ratio of numerator to denominator inputs. Continuous measurement of this potential fall provides the output signal.

The Miller integrator. In order to explain adequately the operation of the circuit, it will be necessary to consider the Miller integrator in more detail. Fig. 1 shows the basic arrangement of the circuit. The valve is switched by a square-wave fed to the suppressor. The rate of run-down of anode potential during the "on" period (when the suppressor is at earth) is inversely proportional to the product of R_{19} and C_5; the run-down begins at a potential E_p equal to that at the cathode of diode D_1. Recovery of the anode* begins when the suppressor is switched to -100 V; the anode may or may not have "bottomed" (i.e. reached the knee of the I_a/V_g characteristic) before this occurs. On recovery the anode is "caught" at potential E_p by diode D_1, after which the valve is quiescent and ready to begin another cycle.

The rate of anode fall is determined by the current available to discharge capacitor C_5. No current can flow to the grid of V_6, since this is slightly negative with respect to the cathode;

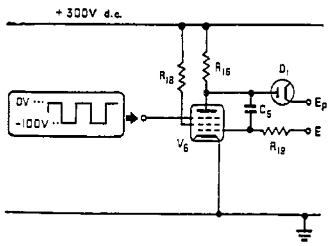

Fig. 1. The Miller integrator

thus the entire discharging current must be provided by R_{19}. If the instantaneous grid potential of V_6 is E_g, the instantaneous current in R_{19} is given by $(E - E_g)/R_{19}$, and this is equal to the current discharging capacitor C_5, i.e. $-C_5[d(E_a - E_g)/dt]$, where E_a is the instantaneous anode potential. Thus

$$-C_5[d(E_a - E_g)/dt] = (E - E_g)/R_{19}$$

and, assuming $E_g = 0$ (which is true to a first approximation), we have

$$dE_a/dt = -E/C_5R_{19} = \text{constant} \qquad (1)$$

if E is constant.

(The effect of the approximation is to give a calculated run-down rate which slightly exceeds the actual rate, and to conceal a slight non-linearity. The departure from linearity can be reduced to $\frac{1}{4}$% or better, rendering it negligible for most purposes; the difference in run-down rate can be corrected by changing the d.c. level of the two input potentials in a manner to be described later.)

The Sanatron. A detailed description of this circuit may be found elsewhere.[(2,4)] In this application, a Miller integrator circuit, similar to that described above, is arranged so that it may be triggered by a short pulse rather than by a square wave. The circuit resembles to some extent a multivibrator and itself maintains the conditions necessary for the run-down until this is terminated by bottoming. When this occurs the circuit almost immediately initiates its own recovery, and subsequently provides a trigger pulse for its next run-down: that is, it "self-runs." A signal is also derived from it to switch the Miller circuit.

Ratio computation with Miller integrators. The rate o linear run-down of a Miller integrator anode is given approxif mately by equation (1). If the anode is allowed to bottom.

* By "anode" is meant, of course, "anode potential"; the author follows here the precedent of Williams[(1)] in preferring the more graphic, if less strictly correct, description.

the total amplitude of run-down is $E_p - E_k$, where E_k is the bottoming potential; the overall time t of run-down is given by

$$(E_p - E_k)/t = + E/C_5R_{19} \quad (2)$$

or

$$t = [C_5R_{19}(E_p - E_k)]/E \quad (3)$$

Suppose now that it is required to compute the ratio E_N/E_D of two potentials E_N and E_D. Let the denominator potential E_D be used as the input E in the above relation (3), so that we have

$$t = [C_5R_{19}(E_p - E_k)]/E_D \quad (4)$$

and let us construct this circuit in the form of a Sanatron (Fig. 2). V_6 is the Miller integrator valve; V_4 is a control valve which feeds a suitable switching wave-form to the

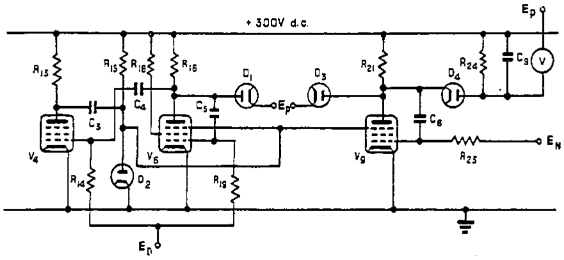

Fig. 2. Basic diagram of ratio computer

suppressor of V_6. The short time constant C_3R_{15} provides for retriggering of the circuit a short time after the completion of recovery.

A second Miller stage V_9 is then constructed. This is switched by the same waveform as is fed to the suppressor of V_6, and thus is able to "run-down" only while V_6 is doing so; it has the same anode "catching" potential E_p, fed by diode D_3. The potential fed to the grid of V_9 is that forming the numerator E_N of the ratio which we wish to compute. The circuit constants are so arranged that the anode of this second stage can never reach "bottoming."

The relation between run-down time and applied potentials for the second stage, by analogy with equation (4), is

$$t = [C_8R_{23}(E_p - E_T)]/E_N \quad (5)$$

where E_T is the potential reached by the anode of V_9 at the moment when V_6 bottoms and both suppressors are returned to -100 V.

Since the run-down times t are the same in the two stages, we have, by equating (4) and (5):

$$(C_5R_{19}/E_D)(E_p - E_k) = (C_8R_{23}/E_N)(E_p - E_T) \quad (6)$$

or since C_5, C_8, R_{19}, R_{23} and $(E_p - E_k)$ are all constant:

$$1/E_D \propto (E_p - E_T)/E_N \quad (7)$$

or

$$E_p - E_T \propto E_N/E_D \quad (8)$$

$E_p - E_T$ represents the *amplitude* of run-down of V_9. If now we measure the final potential reached by the anode of V_9 *from* E_p (its starting potential) *as zero*, we have, calling this quantity E':

$$E' \propto E_N/E_D \quad (9)$$

The method of making this measurement is shown in Fig. 2: a peak rectifier diode D_4 and a high-resistance voltmeter connected between the anode of the diode and potential E_p give explicitly the required quantity, which is proportional to the ratio of the two potentials E_N/E_D. The meter may be scaled so that, when the two potentials are equal, its reading is unity.

CIRCUIT DETAILS

The complete circuit is shown in Fig. 4 and waveforms at various points appear in Fig. 3. The Sanatron comprises valves V_2 to V_6. An additional valve, V_2, has been employed to provide improved automatic self-trigger, since the method shown in Fig. 2 (in which the suppressor of

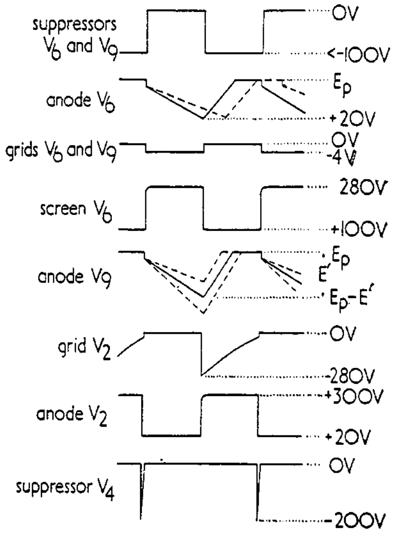

Fig. 3. Waveforms at various points of Figs. 2 and 4

V_6 is merely allowed to rise slowly to h.t. potential) is somewhat indefinite. The improved method depends on the fact that the screen of a Miller valve falls sharply when the anode bottoms. This fall is fed by C_6 and R_9 to the grid of V_2, cutting off the cathode current so that the anode rises sharply to h.t. potential. The grid then attempts to recover exponentially to h.t.; when it reaches earth potential, cathode current again flows and the anode of V_2 falls sharply. This fall is differentiated by the components C_2 and R_{10} and fed as a trigger pulse to the suppressor of V_4. (The positive pulse when V_2 is cut off is not communicated to the suppressor of V_4 owing to the presence of diode V_{3a}.)

V_4 and V_6 form the Sanatron proper. V_4, the control valve, is held off by the run-down at the anode of V_6, which is fed to its grid through C_4; the components R_{14}, C_4 and R_{17} are so chosen that the grid falls slowly until it is caught by diode V_{3b} at about $-6\frac{1}{2}$ V. (This ensures that when the run-down is complete the grid of V_4, rising again on a time constant C_4R_{14} until the valve conducts, has not to rise several tens of volts, thus delaying the recovery needlessly. It may, however, be noted that the unavoidable *slight* delay in the initiation of recovery, which is a disadvantage in some applications of the Sanatron, is of no importance here, since the delay time is always a constant fraction of the run-down time and therefore does not affect the computing accuracy.)

Potential E_p (Figs. 1 and 2) is provided by a neon-stabilizer tube V_7, and is constant at about $+200$ V. Diodes V_{5b} and V_{8a} employ this potential to catch the anodes of V_6 and V_9 respectively; the meter is also connected to it so that the run-down amplitude of V_9 is measured relative to potential E_p. V_9 is the second Miller valve; its grid feed resistor is now

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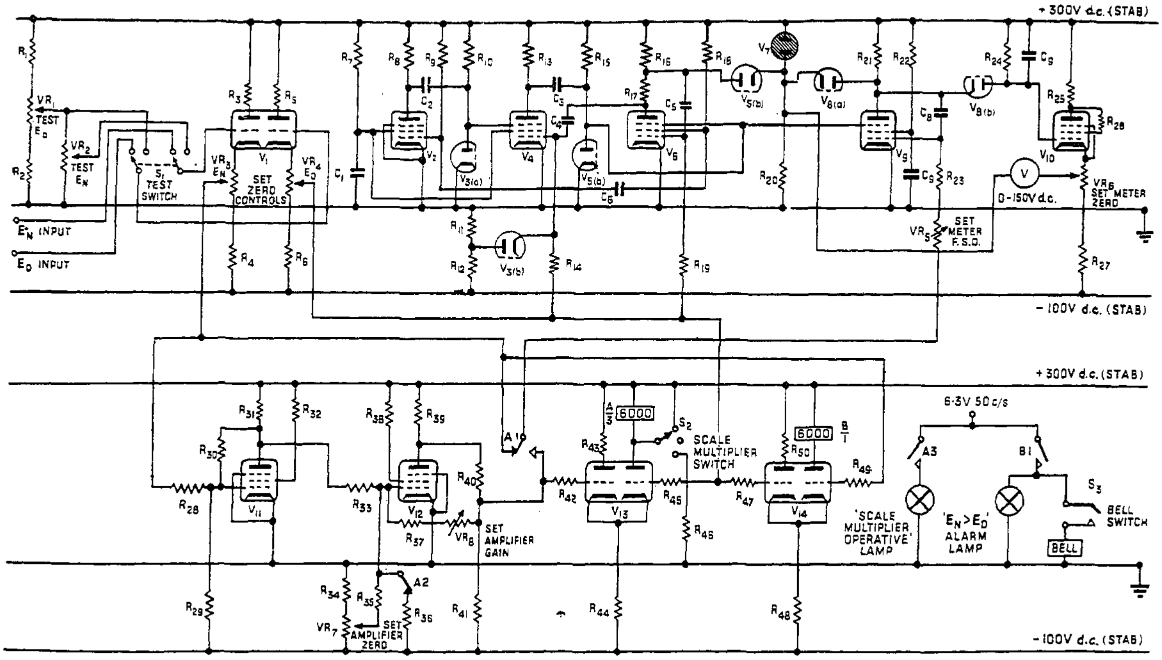

Fig. 4. Complete circuit diagram of the ratio computer

$R_1 = 220\ k\Omega$	$R_{14} = 1{\cdot}5\ M\Omega$	$R_{27} = 39\ k\Omega$	$R_{39} = 100\ k\Omega$	$C_1 = 0{\cdot}25\ \mu F$	$VR_7 = 20\ k\Omega$
$R_2 = 4{\cdot}7\ k\Omega$	$R_{15} = 470\ k\Omega$	$R_{28} = 1\ M\Omega\ (\pm 1\%)$	$R_{40}, R_{41} = 470\ k\Omega$	$C_2 = 100\ \mu\mu F$	$VR_8 = 100\ k\Omega$
$R_3 = 100\ \Omega$	$R_{16} = 100\ k\Omega$	$R_{29} = 1\ M\Omega$	$R_{42} = 4{\cdot}7\ M\Omega$	$C_3 = 0{\cdot}05\ \mu F$	
$R_4 = 33\ k\Omega$	$R_{17} = 4{\cdot}7\ k\Omega$	$R_{30} = 1\ M\Omega\ (\pm 1\%)$	$R_{43} = 47\ \Omega$	$C_4, C_5 = 470\ \mu\mu F$	$V_1 = 6SN7$
$R_5 = 100\ \Omega$	$R_{18} = 68\ k\Omega$	$R_{31} = 100\ k\Omega$	$R_{44} = 22\ k\Omega$	$C_6, C_7 = 0{\cdot}05\ \mu F$	$V_2 = EF50$
$R_6 = 33\ k\Omega$	$R_{19} = 1\ M\Omega$	$R_{32} = 220\ k\Omega$	$R_{45} = 4{\cdot}7\ M\Omega$	$C_8 = 470\ \mu\mu F$	$V_3 = 6H6$
$R_7 = 22\ k\Omega$	$R_{20} = 47\ k\Omega$	$R_{33} = 220\ k\Omega\ (\pm 1\%)$	$R_{46} = 220\ k\Omega$	$C_9 = 0{\cdot}1\ \mu F$	$V_4 = EF50$
$R_8 = 220\ k\Omega$	$R_{21} = 100\ k\Omega$	$R_{34} = 82\ k\Omega$	$R_{47} = 4{\cdot}7\ M\Omega$		$V_5 = 6H6$
$R_9 = 330\ k\Omega$	$R_{22} = 47\ k\Omega$	$R_{35} = 270\ k\Omega$	$R_{48} = 22\ k\Omega$	$VR_1 = 20\ k\Omega$	$V_6 = EF50$
$R_{10} = 470\ k\Omega$	$R_{23} = 470\ k\Omega$	$R_{36} = 10\ M\Omega$	$R_{49} = 4{\cdot}7\ M\Omega$	$VR_2 = 100\ k\Omega$	$V_7 = 7475$
$R_{11} = 6{\cdot}8\ k\Omega$	$R_{24} = 10\ M\Omega$	$R_{37} = 390\ k\Omega\ (\pm 1\%)$	$R_{50} = 47\ \Omega$	$VR_3, VR_4 = 5\ k\Omega$	$V_8 = 6H6$
$R_{12}, R_{13} = 100\ k\Omega$	$R_{25}, R_{26} = 47\ \Omega$	$R_{38} = 220\ k\Omega$		$VR_5 = 1\ M\Omega$	V_9–$V_{12} = EF50$
				$VR_6 = 5\ k\Omega$	$V_{13}, V_{14} = 6SN7$

made variable (R_{23} and VR_5) to enable the meter deflexion to be set at unity when the two input voltages are equal.

The peak rectifier diode V_{8b} now works into a cathode follower V_{10} rather than directly into the meter, to avoid the latter loading the anode of V_9 unduly. A potentiometer VR_6 in the cathode circuit of V_{10} enables the meter to be set at zero deflexion when no input is present.

Level transformation of input signals. In the analysis of the Miller circuit the assumption was made that the grid potential is zero during the run-down. This approximation becomes increasingly invalid as the input potentials become smaller; its effects can be avoided if the input potentials are measured with respect to *the grid potential of the valve to which they are fed*, rather than with respect to earth. V_1 achieves the necessary shift in level of the two signals; the levels may be adjusted by potentiometers VR_3 and VR_4.

Automatic scale multiplier. It is frequently found convenient, for example in multi-range test meters, to provide a press button which doubles the meter deflexion. In this device such a facility has been made automatic, and is provided by valves $V_{11} - V_{13}$ and relay A.

The numerator input potential E_N (modified by V_1) is fed continuously to the input of a two-stage d.c. amplifier or "anode follower," V_{11} and V_{12}. This is so designed that its gain can be adjusted to exactly 2, and that its output (junction of R_{40} and R_{41}) can be made equal to the grid potential of V_9 for zero input. The output is fed to one grid of V_{13}, the other grid of which is connected directly to the modified denominator potential E_D. Relay A is connected in one anode of V_{13}. It will be seen that, if $2E_N > E_D$, relay A will be released. In this condition contact $A1$ connects E_N directly to the grid feed resistors R_{23} and VR_5 of V_9. If, however, $2E_N < E_D$, relay A is operated and contact $A1$ connects $2E_N$ to the grid input of V_9. The deflexion of the meter is accordingly doubled. Contact $A3$ lights a warning lamp to show that the doubling circuit is in operation.

Contact $A2$ applies a form of electrical backlash to the circuit. When relay A is released, contact $A2$ connects resistor R_{36} to the grid of V_{12}; this makes the output of V_{12} slightly more positive than it would otherwise be, causing the discriminator to "trip" at 46% of full-scale deflexion rather than at 50%. The operation of relay A opens contact $A2$, and the discriminator returns to its previous state at 50%. A 4% backlash is thus introduced which prevents chattering of the relay. The scale-doubling facility may be rendered inoperative or permanently switched into circuit by setting switch S_2 to the appropriate position.

$E_N > E_d$ alarm. It is clear that the circuit cannot give reliable results if V_9 is allowed to bottom. In the present design this occurs when $E_N > E_D$; some audible warning of its occurrence was thought desirable. V_{14} provides this facility. Its general principle is similar to that of V_{13}, but in this case relay B operates if $E_N > E_D$ and contact B1 lights an alarm lamp and (provided S_3 is closed) rings a bell.

Adjustment. To facilitate adjustment of the device a test switch, S_1, and two test potentiometers, VR_1 and VR_2, have been provided. It will be seen that the test E_N signal is derived from the test E_D signal by a potentiometer, so that if

VR_2 is held stationary and VR_1 is moved over a range of voltage the deflexion of the meter should not alter if the circuit is functioning correctly. By employing this method of aligning the circuit it is possible to avoid the use of an oscillograph or additional meter. It has also the advantage that the circuit is tested in the most stringent way possible—by two potentials which are *known* to be in a constant ratio to each other.

PERFORMANCE

The accuracy of the device can easily be checked by use of the "test" controls. In this laboratory, where the input signals were $0 \leqslant E_N \leqslant 70$ V and $15 \leqslant E_D \leqslant 70$ V respectively, an accuracy of the order of $\frac{1}{2}\%$ of full-scale deflexion was obtained over the full ranges of both E_N and E_D.

The overall response time of the circuit is governed by that of the peak rectifier, which must be made sluggish compared with the cyclic time of the Miller circuits. With the component values given, a step function rising signal is reproduced as an exponential of time constant about 1/20 sec (the response of the meter itself), and a falling signal with a time constant of about 1 sec, which is adequately short for most applications. If, however, the Miller circuits are operated at a higher recurrence frequency the peak rectifier response time (and hence the overall response time) can be reduced. It is perhaps in its potential speed of response that this circuit has its greatest advantage over the servo-mechanism type of computer. Useful meter readings cannot of course be obtained at (say) 25 c/s, but pen recorders exist capable of recording sinusoidal signals up to 50 c/s. The output arrangements would need to be modified appropriately.

APPLICATIONS

Analogue computing. The circuit as given here was designed for a restricted range (approximately 4 : 1) of E_D. There is no reason why a considerably extended range should not be covered, but the accuracy falls off at small values of E_D. Further, since the recurrence frequency of the circuit is roughly proportional to E_C, changes may be needed to maintain correct circuit operation when E_D and the recurrence frequency are low.

The same circuit principles may be applied to the computation of reciprocals and products. Reciprocals can be dealt with directly by the present circuit, for if E_N is held constant at some convenient value the output is inversely proportional to E_D. A skeleton circuit for product computation is shown in Fig. 5. V_1 is, as before, the Miller valve of a Sanatron.

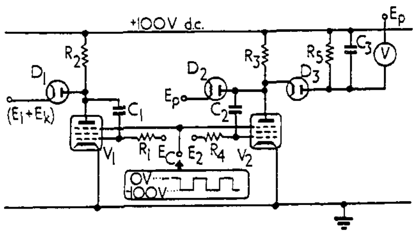

Fig. 5. A skeleton circuit for the computation of products

V_2 is a second Miller valve switched in parallel with V_1. The grid feed resistor of V_1 is now connected to a constant potential, E_c, and the anode catching diode D_1 to a function $(E_1 + E_k)$ of the first input potential E_1, where E_k is the potential, relative to earth, at which V_1 bottoms. (This potential $(E_1 + E_k)$ can be simply produced by d.c. amplification.) The second input potential E_2 is fed, as before, to the grid resistor of V_2. Consideration of equations (4) and (5), revised appropriately, shows that the output potential is now proportional to the product E_1E_2.

Applications to monitoring of radiation sources and similar uses. In a number of spectrographic instruments, and in microdensitometry, it is necessary to measure the ratio of the radiation intensity transmitted along two different optical paths. This has generally been carried out by the use of a self-balancing potentiometric recorder, which is an expensive solution where recording is not otherwise needed. The present circuit enables a single direct-reading meter to be employed with these instruments instead of the recorder, provided that the intensities to be measured can be converted into suitable steady potentials.

The particular application for which the computer was designed illustrates the type of application to which it is suited. In the measurement of X-ray diffraction intensities by a Geiger–Müller counter it is often necessary to monitor continuously, by means of a second counter, the characteristic ($K\alpha$) radiation emitted by the X-ray tube; a measure of the absolute intensity of a diffraction line is obtained by dividing the line intensity by the tube intensity as indicated by the monitor counter. Counting-rate meters are used to obtain a steady potential proportional to the X-ray intensity striking each counter; this potential (in the case of the Atomic Energy Research Establishment rate-meter type 1037A, the circuit[7] of which has been used in this laboratory with slight modifications) is suitable to be fed directly into the computer.

CONCLUSION

A circuit has been described for computation of the ratio of two steady or slowly-varying potentials. The possibility of its extension to computation of reciprocals or products has been mentioned. The performance of the circuit cannot rival for accuracy that of the best servo-mechanisms, but it may be a simple and useful substitute where the highest accuracy is not required; its speed of operation, however, can be made considerably faster than that of a mechanical servo.

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